Candidate:  

Grade Level(s): 6  

Date Taught:  

Content Area: Math  

Topic: Greatest Common Factor  

Lesson Duration: 30 minutes  

Teacher Materials: Projector  

Student Materials: Worksheet, popsicle sticks, colored pencils, pencils, butcher paper  

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**A1. Lesson Pre-Plan**

Provide a description of your students (IEPs, EBs, cultural backgrounds, interests, etc.):
- Four of my students have IEP’s and two of my students are Emergent Bilinguals. Some supports that seem to help these students are sentence frames, purposeful manipulatives, and ample wait time when posed a question. The majority of my students are Caucasian, and about 16 of my 27 students are under the proficiency expectation in math based on their MAP testing scores. There are two students who love math in my class, but the majority of the class do not seem very enthusiastic about this subject.

What prior knowledge will students bring to this lesson?
- My students have just been introduced to the greatest common factor using the more procedural method. They have some knowledge of the GCF which should be helpful to them in this lesson. Additionally, they have been practicing highlighting the most important pieces of information when given a word problem, which should help them with their problem attack. They also just finished a unit on prime factorization, so they have experience finding factors, as they were just tested on this information.

How will technology be utilized in this lesson?
- I will project the worksheet onto the projector when giving directions.

Explain the rationale for choosing this lesson:
- I chose this lesson because students are finding the greatest common factor without even knowing that’s what they’re doing. I think it develops a really strong conceptual understanding of the GCF, because students can visually see the groups they’re manipulating. My students have done a problem like this before with arranging tables and chairs, and a lot of them got the right answer, but had no idea what their answer represented in the context of the problem. This is why I think having them use manipulatives will really strengthen their overall understanding. Additionally, it gives them more practice extracting key pieces of information from a problem and using it to find a solution which will really help them in their school careers.

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**A2. Outcomes & A4. Student Assessment**

Content Standards Addressed: 6.NS. 4: **Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.** For example, express 36 + 8 as 4 (9 + 2).

Standard for Mathematical Practice: CCSS.MATH.PRACTICE.MP5 Use appropriate tools strategically.
- CCSS.MATH.PRACTICE.MP2 Reason abstractly and quantitatively.

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Academic Language Demands:
- Vocabulary:
  - factors are numbers we can multiply together to get another number.
  - common factors: factors that are the same between two numbers
  - greatest common factor: the largest of any of those common factors
Lesson Plan Example with Rubric

- **Language Functions-** Explain
- **Related Discourse Forms-** Students should be able to justify their visual responses using the greatest common factor.
- **Language Supports-** I will provide a sentence frame for students with the word problem to help guide their response. I will also give them a list of phrases they can try using when working with a partner, so that they have choice in how they want to express themselves.

**Sentence frame for activity:**
“There are _____ baskets total. In each basket, there are ______ oranges, ______ apples, and ________ bananas.”

**Sentence frame for working with a partner:**
“I think ________, because __________.”
“I agree with you, and I also believe that __________.”
“I heard you say ________. That makes me think __________.”
“Can you show me how you found the information about ________?”
“Why do you think that?”
“I heard you say ________. I disagree because __________.”

**English Language Development:**
- **ELD Standard(s)-** 6. Part 1. A. 1. Bridging. Contribute to class, group, and partner discussions by following turn-taking rules, asking relevant questions, affirming others, adding relevant information, paraphrasing key ideas, building on responses, and providing useful feedback.
- **EDL Objective(s)-** When problem solving, students should collaborate and use their language to defend their ideas and inquire into the ideas of others. Additionally, students will be able to explain “why” their visual manipulatives represent the greatest common factor.

**Specific Instructional Outcomes (Objectives):**
Students should be able to use the greatest common factor to arrange objects in a way that there are none leftover.

**Student-Friendly Objectives:**
Students should be able to arrange objects in identical groups so that there are none leftover, using the GCF.

**Assessment(s) of Outcomes: (include formal & informal)**
Informally, I will walk around and assess student understanding by seeing how they work with their partners. I will evaluate which students are engaging in more of the explaining role, and note any students who seem to be struggling. Formally, I will grade the worksheets and note areas of difficulty and misunderstanding.

**A3. Designing Coherent Instruction**

<table>
<thead>
<tr>
<th>Time</th>
<th>Procedure</th>
<th>NOTES</th>
</tr>
</thead>
</table>
| (e.g., 9:00-9:15 am) | **Before:** I will intro the activity saying that we will be working in small groups, so it is important that we follow our small group norms (which are posted on the classroom wall). This is when I will show students the sentence frames they can use to respectfully work with their peers:  
“I think________, because __________.” | Identify supports for Emergent Bilinguals and Students with Disabilities (A5 & A6), as well as opportunities for multiple means of Engagement, Representation, and Expression (UDL) By purposefully choosing who my Emergent Bilinguals and Students with Disabilities are grouped with, this should help them be most successful with this activity. Because I have two |
“I agree with you, and I also believe that __________.”
“I heard you say __________. That makes me think __________.”
“Can you show me how you found the information about __________?”
“Why do you think that?”
“I heard you say __________. I disagree because __________.”

With this activity, I really don’t want to give too much away because I want them to explore on their own and with their peers. As a result, I think before I throw them into the activity, I will review the fruit basket problem challenge as a class:

**Prompt:** “Local food pantries are assembling food baskets for donation. Determine how many baskets volunteers can make using the given fruit, so that each basket contains the same number of each kind of fruit. The goal is to make as many identical baskets as possible, with no fruit leftover. Use colored markers or pencils to **model your work** on the piece of butcher paper, and show what should appear in a basket, or **explain** how you found your answer.”

As I read aloud, I will have my students highlight the important information from the problem. Afterwards, I will have them share as a class, and I will highlight on the projector the appropriate information they need to solve the problem. For example, in this problem they should arguably highlight: “determine how many baskets,” “identical baskets,” “no fruit leftover,” and “model your work.” These phrases help indicate that they are finding the GCF, and also tell them how they should show their work. I will also tell my students that they are free to use their multiplication tables to help them solve their problem. They’ve used multiplication tables before to find common factors, so this should activate their prior knowledge and get them thinking about using factors to solve this problem.

Afterwards, I will call out the assigned groups, and have them each pick up a worksheet, set of popsicle sticks, butcher paper, and colored pencils. I really want to avoid telling kids we’re finding the greatest common factor at the beginning of the lesson, and have them discover this themselves.

### 35 minutes

**During:** Students will work on a worksheet (attached at end) with a partner or group of three. They will use their popsicle sticks to represent the fruit in the basket arrangements. (e.g. one orange colored popsicle stick = one orange). They can make groupings of popsicle sticks so that they can visually see how many baskets they’re making, and how much fruit is in each one. Additionally, they will have one piece of butcher paper per group where they will draw their arrangements for each problem. At this point, I will walk around the room and help assist groups who may be struggling. I predict that there may some groups who find factors, but not the greatest common factor. At this point, I will refer them to the multiplication chart and see if they can figure out a way that they could get a greater amount of groups. If they’re still struggling, I will have them talk to a group who I know has the correct answer and can explain their thinking.

See multiplication chart on next page: My students already have this in their Instructional Student Notebook (ISN), so they can pull this out at this time. They also know how to use it to find factors for a given number.

**Spanish speakers:** I’m considering pairing them together, and allowing them to speak in their L1 if they decide to. For my one student on the spectrum, I will pair him with people he generally works well with, so he feels comfortable sharing his very intelligent ideas. Lastly, for my students who go to Resource for math, I will pair them with middle level students and keep an eye on them so I can ask them probing questions if they seem stuck or unsure.

Additionally, I will provide students sentence frames for how they can respectfully interact with their peers and express their ideas.

I will also allow students to pull out their multiplication tables at this time to help them find factors if they don’t already know them mentally.

**Students will have options in how they want to represent their findings. They can 1) use the popsicle sticks and then draw their groupings onto the butcher paper, 2) Draw the configurations in their notebooks and then transfer it to the butcher paper, or 3) Find the GCF using numbers (without any visual representation), and then express what they’re numbers mean on the piece of butcher paper by creating a visual.**
Lesson Plan Example with Rubric

15 minutes

**After:**

During this time, I want to ask a couple of different groups to share their strategies for solving the fruit basket challenge. I will remind them of the purpose of this lesson: to use different strategies to conceptually understand how to find the greatest number of identical groups. I will be purposeful in which groups I have speak, because I want to ensure that different strategies are shared (i.e. using popsicle sticks to represent the groupings, drawing the groups and then transferring it to butcher paper, using a more procedural method and then conceptualizing it, etc.) As students present their work, these are some questions I might ask:

- How did you know you had the maximum amount of groups represented?
- When did you make the connection to use GCF? Did you know from the very beginning? If so, how? If not, when did you discover this?
- How did you use the manipulatives to represent the maximum amount of groups?
- How did drawing the groupings on butcher paper help you to understand the concept of GCF?

At this point, I’m hoping that students will have made the connection to finding the greatest common factor. If not, I will ask the class, “By finding the greatest number of identical baskets of fruit, what math concept from sixth grade have you been using? Go ahead and turn and talk to a partner.” After responding to comments, I will say, “Today we found the GCF but not by just using the factor tree, step diagram, or factor pairs. While some of you may have used these methods, we all visually represented our groupings on our posters. It’s incredibly important in math to not just do a procedure without fully

By exposing students to multiple strategies of solving the challenge, this is inherently UDL, because it shows them that there are different ways to perceive a problem. This opens their eyes to different ways of thinking.
| Understanding the meaning behind it. By using drawings and manipulatives we were able to physically see what finding the greatest common factor allows us to do. Turn and talk with a partner about what the greatest common factor is useful for.” Then afterwards, I will repeat and rephrase student responses and say something like: “Very good! Using the greatest common factor allows us to use all of our resources to create the maximum amount of identical groups. This can be incredibly helpful in party planning, when planners have to figure out how many tables and chairs they need. You can also use the GCF if you want to make party favors for your guests, with the same number of goodies inside each bag.”

| “Using the methods you’ve already learned to find the GCF are great and can be incredibly efficient. However, I want to encourage you to also really think about what the problem is asking and visualize what you need to do. And when you find the answer, think to yourself, “Does this make sense? Does this fully and appropriately answer the question?” Engaging in this type of thinking will make you all better math students and will help you be successful when even more challenging math comes your way next year in seventh grade!”

| At this point, I will ask students to complete an exit ticket on their own:

| Ms. Umansky is making Halloween goodie bags for her sixth-grade students. She has 60 Reese’s Peanut Butter Cups, 45 Milky Ways, and 30 Kit Kats. What is the greatest number of goodie bags she can make if each bag is made identically? How many Reese’s Peanut Butter Cups, Milky Ways, and Kit Kats will be in each bag? If she has 30 students, does she have enough goodie bags? Why or why not? | My students in Resource or math intervention may need more time, so they can work on their exit ticket when they’re pulled out for math help. However, I will tell the Resource and math intervention instructors that I would first really like them to try this problem on their own. If they are struggling, then I would have them intervene and provide help. (I would also have the instructors report back to me whether the students were able to do it by themselves or if they needed help—and to what extent). |
THE FRUIT BASKET CHALLENGE

Local food pantries are assembling food baskets for donation. Determine how many baskets volunteers can make using the given fruit, so that each basket contains the same number of each kind of fruit. The goal is to make as many identical baskets as possible, with no fruit leftover. Use colored markers or pencils to model your work, and show what should appear in a basket.

Example: 12 apples and 8 oranges
Answer: 4 baskets total. Each basket contains 3 apples and 2 oranges. Draw 3 red circles for the apples and draw 2 orange circles for the oranges.

1. 16 apples and 8 oranges
2. 12 apples and 9 oranges
3. 20 apples and 15 oranges
4. 7 apples and 13 oranges
5. 7 apples and 14 oranges
6. 8 apples and 22 oranges
7. 6 apples, 8 oranges, and 10 bananas
8. 3 apples, 6 oranges, and 12 bananas
9. 10 apples, 15 oranges, and 25 bananas
10. 10 apples, 10 oranges, and 16 bananas
11. 9 apples, 15 oranges, and 6 bananas
12. 4 apples, 8 oranges, and 17 bananas
13. 4 apples, 6 oranges, 8 bananas, and 12 pears
14. 6 apples, 12 oranges, 15 bananas, and 9 pears
15. 3 apples, 3 oranges, 3 bananas, and 3 pears
16. 8 apples, 14 oranges, 4 bananas, and 8 pears
17. 9 apples, 6 oranges, 12 bananas, and 18 pears
18. 5 apples, 10 oranges, 15 bananas, and 10 pears
19. What patterns did you notice?
20. Did you have an answer in which just 1 fruit basket could be made? Why?
21. How did you decide how to group your items?
### A1. KNOWLEDGE of STUDENTS (Ss)  **TPE 1, UDL**  **SCORE: 3**

<table>
<thead>
<tr>
<th>Not Demonstrated – 1</th>
<th>Partially Demonstrated – 2</th>
<th>Demonstrated – 3</th>
<th>Demonstrated with Distinction – 4</th>
</tr>
</thead>
</table>
| ● TC shows **minimal** understanding of how Ss learn – in their ways of learning, knowledge & skills, special needs, interests, & cultural identities – and **does not indicate** that such knowledge is valuable | ● TC shows **general** knowledge and awareness of how Ss learn – in their ways of learning, knowledge & skills, special needs, interests, & cultural identities – but plans to teach to the **class as a whole** | ● **TC purposefully acquires** knowledge about how the **whole class AND groups** of Ss learn – in their ways of learning, knowledge & skills, special needs, interests, & cultural identities – and **plans lessons accordingly**  
● **TC understands** that Ss learn through developmentally appropriate & active intellectual engagement with content  
● Utilizes needs and abilities to plan lesson  
● Students actively involved in learning (e.g., use of manipulatives)  
● In notes section of A3, the TC shows specific accommodations they are making for various groups of students | ● **TC purposefully acquires** knowledge about how the **whole class, groups, AND individual** Ss learn – in their ways of learning, knowledge & skills, special needs, interests, & cultural identities – and plans lessons accordingly  
● **TC understands** that Ss learn through developmentally appropriate & active intellectual engagement, **AND that misconceptions & gaps** in knowledge and experience may need to be uncovered |
### A2. SETTING INSTRUCTIONAL OUTCOMES

<table>
<thead>
<tr>
<th><strong>Not Demonstrated – 1</strong></th>
<th><strong>Partially Demonstrated – 2</strong></th>
<th><strong>Demonstrated – 3</strong></th>
<th><strong>Demonstrated with Distinction – 4</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>• Outcomes represent <strong>low</strong> expectations/lack of rigor</td>
<td>• Outcomes represent <strong>moderate</strong> expectations/ri gor</td>
<td>• <strong>Most</strong> outcomes represent high expectations/ri gor</td>
<td>• <strong>All</strong> outcomes represent high expectations/ri gor</td>
</tr>
<tr>
<td>• All outcomes are <strong>unclear</strong></td>
<td>• Some outcomes are <strong>unclear</strong></td>
<td>• <strong>Most</strong> outcomes are clear</td>
<td>• <strong>All outcomes are clear</strong></td>
</tr>
<tr>
<td>• Outcomes are <strong>not measurable</strong></td>
<td>• Some outcomes are <strong>not measurable</strong></td>
<td>• <strong>Most outcomes are measurable</strong></td>
<td>• <strong>All outcomes are measurable</strong></td>
</tr>
<tr>
<td>• Outcomes are <strong>poorly aligned</strong> with content standards</td>
<td>• Outcomes are <strong>somewhat aligned</strong> with content standards</td>
<td>• <strong>Most outcomes are aligned</strong> with content standards</td>
<td>• <strong>All outcomes are aligned</strong> with content standards</td>
</tr>
<tr>
<td>• All outcomes are <strong>not suitable</strong> for most Ss</td>
<td>• Some outcomes are <strong>suitable</strong> for most Ss</td>
<td>• <strong>Most outcomes are suitable</strong> for most Ss</td>
<td>• <strong>All outcomes are suitable</strong> for Ss with differentiation/flexibility for individual Ss</td>
</tr>
</tbody>
</table>

- The outcomes do not specifically state level of mastery expected
- The learning outcomes focus on understanding of the meaning of _______ (i.e., greatest common factor); Ss are asked to explain thinking
- The outcomes are clear and directly align with the content standard
- The outcomes were designed in a flexible way (e.g., use of manipulatives, drawings, or procedures); Ss are given the opportunity to speak in their L1
A3. DESIGNING COHERENT INSTRUCTION  

**SCORE: 3**

<table>
<thead>
<tr>
<th>Not Demonstrated – 1</th>
<th>Partially Demonstrated – 2</th>
<th>Demonstrated – 3</th>
<th>Demonstrated with Distinction – 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Learning activities are poorly aligned with learning outcomes and/or content standards</td>
<td>Some learning activities are aligned with learning outcomes and/or content standards</td>
<td>Most learning activities are aligned with learning outcomes and content standards</td>
<td>All learning activities are aligned with learning outcomes and content standards</td>
</tr>
<tr>
<td>Learning activities do not follow an organized progression</td>
<td>Some learning activities do not follow an organized progression</td>
<td>Most learning activities follow an organized progression</td>
<td>All learning activities follow an organized progression</td>
</tr>
<tr>
<td>Ss are not actively engaged in cognitive activities with no evidence of UDL</td>
<td>Ss are minimally engaged in cognitive activities with limited evidence of UDL</td>
<td>Ss are engaged in cognitive activities with evidence of UDL</td>
<td>Ss are challenged in high-level cognitive activities with appropriate UDL</td>
</tr>
<tr>
<td>No use of groupings</td>
<td>Some use of groupings but may be inappropriate</td>
<td>Appropriate use of groupings</td>
<td>Appropriate and varied groupings that include S choice</td>
</tr>
<tr>
<td>Unrealistic time allocations</td>
<td>Uneven time allocations</td>
<td>Appropriate time allocations</td>
<td>Appropriate time allocations with flexibility for individual Ss</td>
</tr>
</tbody>
</table>

- Evidence of UDL through S choice of representation
- Time allocations appropriate with ________ minutes for Ss to work and ________ minutes for discussion
- All activities are directly tied to GCF and discussing what it represents in real-life situations; Ss are given the opportunity to focus on both the content standard and ELD outcomes
- The activities follow an organized progression
- The groupings are appropriate (e.g., small groups, purposeful pairing of EBs and S w/ IEP).
### A4. DESIGNING STUDENT ASSESSMENT

**TPE 5**  
**SCORE: 3**

<table>
<thead>
<tr>
<th>Not Demonstrated – 1</th>
<th>Partially Demonstrated – 2</th>
<th>Demonstrated – 3</th>
<th>Demonstrated with Distinction – 4</th>
</tr>
</thead>
</table>
| ● Formative assessments do not match learning outcomes and/or content standards  
● Lack of criteria for expectations  
● Minimal formative assessment | ● Formative assessments partially match learning outcomes and/or content standards  
● Criteria available but unclear  
● Rudimentary use of formative assessment | ● Formative assessments match learning outcomes and content standards  
● Criteria clear  
● Appropriately-designed formative assessment | ● Formative assessments clearly match learning outcomes and content standards  
● Well-developed criteria  
● Well-designed formative assessment adapted to individuals as needed.  
● Ss contribute to assessment process |
| | ● Outcome stated but unclear as to how Ss will demonstrate/articulate this | ● Whole class activity, group work, and exit ticket all match learning outcomes and content standards  
● Formative assessment involves monitoring group progress | |
**AS. SUPPORTING EMERGENT BILINGUALS**  

**TPE 1**  

**SCORE: 3**

<table>
<thead>
<tr>
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<th>Demonstrated with Distinction – 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Missing or inappropriate ELD standards</td>
<td>• ELD standards not aligned with learning outcomes, assessments, and/or instructional activities</td>
<td>• ELD standards aligned with &amp; support learning outcomes, assessments, &amp; instructional activities</td>
<td>• ELD standards aligned with &amp; support learning outcomes, assessments, &amp; instructional activities AND meet specific needs of individual ELs/EBs</td>
</tr>
<tr>
<td>• No attempt to draw on home language, culture, and/or prior knowledge</td>
<td>• Limited or superficial attempts to draw on home language, culture, and/or prior knowledge</td>
<td>• Some attempt to draw on home language, culture, and/or prior knowledge</td>
<td>• Tasks draw on home language, culture, AND prior knowledge</td>
</tr>
<tr>
<td>• Missing or inappropriate language supports or instructional scaffolds to engage EBs</td>
<td>• Few language supports and instructional scaffolds to engage EBs</td>
<td>• Whole-class language supports and instructional scaffolds that adequately support EBs' academic language production &amp; content engagement</td>
<td>• Targeted language supports and instructional scaffolds that support individual EBs' academic language production &amp; content engagement</td>
</tr>
</tbody>
</table>

- The ELD standards aligned w/ learning outcome; last activity where groups share their strategies for solving the fruit basket challenge directly supports bridging “contribute to class, group, and partner discussion…”
- TC pairs Spanish speakers together so they can talk in their home language
- Plan indicates whole class support by having Ss highlight key words and phrases as the TC reads the problem
- The sharing of ideas in a group setting supports EBs’ language production and content engagement
- TC provides sentence frames to encourage respectful dialog when sharing ideas
### Supporting Students with Disabilities

**A6. Supporting Students with Disabilities**

**Score: 2**

<table>
<thead>
<tr>
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<th>Demonstrated with Distinction – 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Plans:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>● Lack appropriate instructional methods and/or supports for providing instruction to Ss with disabilities</td>
<td>● Include instructional methods and/or supports that do <strong>not consistently</strong> address the individualized needs of Ss with disabilities</td>
<td>● Include <strong>research or evidence-based</strong> instructional methods and/or supports, including <strong>assistive technology</strong> as needed to support the individualized needs of Ss with disabilities</td>
<td>● Include opportunities for Ss with disabilities to <strong>manage their own</strong> scaffolds and supports, including <strong>assistive technology</strong> as needed</td>
</tr>
<tr>
<td>● Demonstrate <strong>no evidence</strong> of providing accommodations or modifications as stated on student IEPs or 504 plans</td>
<td>● Include <strong>non-individualized</strong> accommodations or modifications OR <strong>do not include</strong> all of the relevant accommodations or modifications as stated on Ss’ IEPs or 504 plans</td>
<td>● Fully address accommodations and modifications as indicated on Ss’ IEPs or 504 plans</td>
<td>● Fully address accommodations and modifications as indicated on Ss’ IEPs and 504 plans and provide opportunities for Ss to understand and advocate for strategies that meet their individual learning needs</td>
</tr>
<tr>
<td>● Do not allow Ss with disabilities access to grade aligned and standards-based instruction</td>
<td>● Allow Ss with disabilities <strong>inconsistent</strong> access to grade aligned and standards-based instruction</td>
<td>● Allow Ss with disabilities access to grade aligned and standards-based instruction</td>
<td>● Allow Ss with disabilities meaningful and consistent access to grade aligned and standards-based instruction</td>
</tr>
<tr>
<td>● TCs instructional supports for 4 students with an IEP include intentional grouping, sentence frames, purposeful manipulatives, ample wait time, choice, resources (such as multiplication tables), multiple ways to represent findings/thinking, extended time, and TC collaboration with resource teacher; these instructional supports are targeted more broadly at all Ss with an IEP rather than individualized supports (with the exception of mentioning the S on the spectrum).</td>
<td>● All Ss are working toward the same learning outcome and standard that is grade aligned</td>
<td></td>
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</tbody>
</table>