

Course Outline for MATH 4531, Differential Geometry

MATH 4531 Differential Geometry (3 units)

Prerequisite: MATH 241 or [MATH 2263](#), and one of the following: MATH 206, MATH 244, [MATH 1151](#), or [MATH 2341](#); or graduate standing in Mathematics.

The theory of curves and surfaces in Euclidean space. May include curvature and torsion of curves, Frenet formulas, covariant derivatives, Cartan structural equations, shape operator, curvature of surfaces, geodesics, Gauss-Bonnet Theorem, and geometric surfaces. 3 lectures. Formerly MATH 404.

Course Learning Objectives	
1	Reparametrize curves in terms of arclength.
2	Compute the curvature and torsion of curves in Euclidean space.
3	Compute covariant derivatives.
4	Compute the principal, normal, and geodesic curvatures of surfaces in Euclidean space.
5	Compute the shape operator (or second fundamental form), and the Gaussian and mean curvatures of surfaces in Euclidean space.
6	Classify curves and surfaces based upon curvature.

Expanded Course Content

Textbooks, materials, and/or other resources
Elementary Differential Geometry, by Barrett O'Neill

Week	Readings	Topics of Discussion
1	Sections 1.1, 1.2, 1.3	Euclidean Space, Tangent Vectors, Directional Derivatives
2	Sections 1.4, 1.5	Curves in $\mathbb{R}^3$, 1-Forms
3	Sections 1.5, 1.6	1-Forms, Differential Forms
4	Sections 1.7, 2.1	Mappings, Dot Product
5	Sections 2.2, 2.3	Curves, The Frenet Formulas
6	Sections 2.3, 3.5, 2.4	The Frenet Formulas, The Congruence of Curves, Arbitrary Speed Curves

Week	Readings	Topics of Discussion
7	Sections 2.5, 2.6, 2.7	Covariant Derivatives, Frame Fields, Connection Forms
8	Sections 2.8, 4.1, 4.2, 4.3, 5.1	The Structural Equations, Surfaces in $\mathbb{R}^3$, Patch Computations, Differentiable Functions and Tangent Vectors, The Shape Operator
9	Sections 5.1, 5.2	The Shape Operator, Normal Curvature
10	Sections 5.3, 5.4	Gaussian Curvature, Computational Techniques
11	Sections 5.4, 6.1, 6.2	Computational Techniques, The Fundamental Equations, Form Computations
12	Sections 6.2, 6.3	Form Computations, Some Global Theorems
13	Sections 6.3, 6.4, 6.5	Some Global Theorems, Isometries and Local Isometries, Intrinsic Geometry of Surfaces in $\mathbb{R}^3$
14	Sections 6.5, 7.1, 7.2	Intrinsic Geometry of Surfaces in $\mathbb{R}^3$, Geometric Surfaces, Gaussian Curvature
15	Sections 7.3, 7.4, 7.6	Covariant Derivative, Geodesics, The Gauss-Bonnet Theorem

Method of Assessment

The possible methods of assessment are written individual examinations, quizzes, in-class worksheets and presentations, projects, and homework. One or more midterms and a cumulative final exam are required.

Instructional Mode

The instructional mode for this class is face-to-face. This designation means that the essential academic standards of the course require in-class student engagement, and thus remote instruction is not appropriate.