

Course Outline for MATH 4512, Non-Euclidean Geometry

MATH 4512 Non-Euclidean Geometry (3 units)

Prerequisite: Graduate standing in Mathematics, MATH 442, or [MATH 3511](#).

Non-Euclidean geometries, including spherical, hyperbolic, and projective geometry. Properties of geodesics, parallels, biangles, Saccheri and Lambert quadrilaterals, angle-sum, area, Pythagorean Theorem, trigonometry, duality, perspectivity, and fundamental theorems. 3 lectures. Formerly MATH 443.

Course Learning Objectives	
1	Generalize the ideas of neutral geometry to new contexts.
2	Make and prove conjectures about hyperbolic, spherical, and projective geometries.
3	Compare and contrast properties in Euclidean, hyperbolic, spherical, and projective geometries.
4	Describe the role and importance of the parallel postulate in the historical development of the geometries.
5	Construct lines and figures in the Poincaré or upper- half-disk model of hyperbolic geometry.
6	Develop and implement models of axiomatic systems, including axiomatic systems for affine and projective planes.
7	Construct appropriate definitions in new geometries for familiar ideas in Euclidean geometry (e.g. polygons).

Expanded Course Content

Textbooks, materials, and/or other resources
1. Greenberg, Marvin J. Euclidean and Non-Euclidean Geometries.
2. Wallace, Edward C., and West, Stephen F. Roads to Geometry.
3. Reynolds & Fenton. College Geometry Using GeoGebra.
4. Instructor-prepared workbook and/or reader

Week	Readings	Topics of Discussion
1	Greenberg, Chapter 5 Wallace and West, Chapter 3 (excerpts provided by instructor)	Neutral geometry; the importance and history of the parallel postulate
2	Instructor Notes	Introduction to spherical geometry; lines and straightness on the sphere; early conjectures about spherical lines and polygons
3	Instructor Notes	Triangles on the sphere; triangle congruence theorems
4	Instructor Notes	Similarity and congruence on the sphere; the parallel postulate and spherical geometry
5	Instructor Notes	Circles and areas on the sphere; Girard's formula
6	Instructor Notes	Other explorations, including: spherical Pythagorean theorem, constructions on the sphere, elliptic geometry more generally
7	Reynolds and Fenton, Chapter 11	Introduction to hyperbolic geometry; lines and segments; the Poincaré disk model and measuring distance and angle
8	Reynolds and Fenton, Chapter 11	Hyperbolic triangles and triangle congruence theorems
9	Reynolds and Fenton, Chapter 11	Polygons; parallelism in hyperbolic geometry; angle of parallelism; horoparallel and hyperparallel lines
10	Reynolds and Fenton, Chapter 11	Hyperbolic circles and constructions in the Poincaré disk; similarity and congruence
11	Reynolds and Fenton, Chapter 11	Other explorations, including: the upper half plane model, Pythagorean theorem in the hyperbolic plane, tessellations, or others
12	Reynolds and Fenton, Chapters 7 and 12	Introduction to finite geometries; an axiom system for an affine plane vs. a projective plane

Week	Readings	Topics of Discussion
13	Reynolds and Fenton, Chapters 7 and 12	Duality and proving related theorems; applications to art and perspectivity
14	Reynolds and Fenton, Chapters 7 and 12	Relating affine planes to projective planes; coordinates for finite geometries
15	Reynolds and Fenton, Chapters 7 and 12	Models for the projective plane; Pappus Theorem and Desargues' Theorem

Method of Assessment

The possible methods of assessment are written individual examinations, quizzes, in-class worksheets and presentations, projects, and homework. One or more midterms and a cumulative final exam are required.

Instructional Mode

The instructional mode for this class is face-to-face. This designation means that the essential academic standards of the course require in-class student engagement, and thus remote instruction is not appropriate.