

Course Outline for MATH 4463, Senior Project Seminar

MATH 4463 Senior Project Seminar (3 units)

Prerequisite: One of the following: MATH 412, MATH 481, [MATH 4201](#), or [MATH 4264](#).

Written and oral analysis and presentation of topics in advanced mathematics. This capstone course in mathematics involves work in a range of mathematical fields, including linear algebra, number theory, graph theory, abstract algebra, and analysis. Not open to students with credit in MATH 460 or [MATH 4464](#). 3 seminars. Formerly MATH 459.

Course Learning Objectives	
1	Integrate and synthesize mathematical knowledge to solve advanced problems in mathematics using a variety of different approaches.
2	Apply accumulated skills and knowledge to the study of advanced topics in mathematics.
3	Present written and oral solutions to problems and proofs of theorems.
4	Explore areas of open research in mathematics.
5	Explore open-ended questions and statements in mathematics, and formulate conjectures by gathering evidence for open-ended problems.

Expanded Course Content

Textbooks, materials, and/or other resources
Text to be chosen by the instructor. Possible textbooks include: • D. Paquin, Lattice Point Geometry and The Geometry of Numbers • J. Sally and P. Sally, Jr., Roots to Research: A Vertical Development of Mathematical Problems, American Mathematical Society This sample course outline follows the textbook J. Sally and P. Sally, Jr., Roots to Research: A Vertical Development of Mathematical Problems, American Mathematical Society.

Week	Readings	Topics of Discussion
1	3.1a: Introduction to Lattice Point	Show that there does not exist an equilateral lattice triangle.

Week	Readings	Topics of Discussion
	Geometry 3.1b: Geometric Shapes as Lattice Polygons	Make a conjecture about the values of n for which there exists a regular lattice n -gon. Prove fundamental properties of lattice lines and angles in the plane.
2	3.1c: Embedding Regular Polygons in a Lattice	Show that there exists a regular lattice n -gon in the plane if and only if $\tan\left(\frac{\pi}{n}\right)$ is rational. Use the Rational Root Theorem and the binomial coefficients, and/or roots of unity, to show that the only positive integer greater than or equal to 3 for which $\tan\left(\frac{\pi}{n}\right)$ is rational is $n=4$.
3	3.2: Which Positive Integers Are Areas of Lattice Squares?	Classify the positive integers that are areas of lattice squares. Find the number of lattice bounds on the boundary of a circle with center $(0,0)$ and radius $\sqrt{n}$.
4	4.1: Dissection of a Lattice Polygon Into Lattice Triangles	Show that every n -gon can be dissected into $n-2$ triangles by means of nonintersecting diagonals such that the vertices of the triangles in the dissection are vertices of the polygon. Show that every lattice polygon can be dissected into primitive lattice triangles.
5	4.2: The Algebraic Structure of the Lattice $\mathbb{Z}^2$	Show that the lattice $\mathbb{Z}^2$ is a free $\mathbb{Z}$ -module. Classify $\mathbb{Z}$ -module bases using determinants and geometry.
6	4.3: The Isometry Group of a Lattice	Classify the geometric transformations that preserve lattice structure. Show that the area of a primitive lattice triangle is equal to $\frac{1}{2}$ using the module structure of the lattice.
7	5.1: Pick's Theorem	Prove Pick's Theorem using topological additivity.
8	5.2; Pick's Theorem	Prove Pick's Theorem using graph theory and Euler's Formula for Planar Graphs.
9	5.3: Visible Lattice Points	Use the algebraic structure of the lattice plane and the Sums of Two Squares Theorem to prove that the probability that two integers are relatively prime is $6/\pi^2$.
10	6.1: Applications of Pick's Theorem	Classify lattice polygons with k interior lattice points and n boundary lattice points.
11	6.2: Farey Sequences	Conjecture and prove properties of Farey sequences. Use Farey sequences to prove Dirichlet's Approximation Theorem.
12	7.1: Lattice Points In and On a Circle	Use the algebraic structure of the lattice plane to obtain Gauss' approximation of π .

Week	Readings	Topics of Discussion
13	8.1: Convex Plane Regions and Integer Points	Show that a nonempty, closed, bounded, convex, proper subset of $\mathbb{R}^n$ is the intersection of its supporting halfspaces. Prove Ehrhart's Theorem on the number of lattice points in the interior and boundary of a bounded convex region in the plane.
14	9.1: Minkowski's Theorem in $\mathbb{R}^n$	Prove Minkowski's Theorem for a General Lattice in $\mathbb{R}^n$. Use Minkowski's Theorem to prove the Sum of Two Squares and Sum of Four Squares theorems.
15	10.1: Embedding Regular Plane Polygons as Lattice Polygons in $\mathbb{R}^k$	Classify the regular polygons that can be embedded as lattice polygons in $\mathbb{R}^k$.

Method of Assessment

The possible methods of assessment are written individual examinations, quizzes, in-class worksheets and presentations, projects, and homework. The final assessment for this course will be a final portfolio.

Instructional Mode

The instructional mode for this class is face-to-face. This designation means that the essential academic standards of the course require in-class student engagement, and thus remote instruction is not appropriate.