

Course Outline for MATH 4342, Nonlinear Dynamical Systems

MATH 4342 Nonlinear Dynamical Systems (3 units)

Prerequisite: One of the following: MATH 242, MATH 244, [MATH 2341](#), or [MATH 2343](#), and one of the following: MATH 241, DATA/[MATH 1265](#), or [MATH 2263](#); or graduate standing in Mathematics.

Qualitative theory of ordinary differential equations: Existence and Uniqueness Theorem, phase portraits, limit sets, stability of fixed points and periodic orbits, energy functions, Poincare-Bendixson Theorem, Poincare maps, bifurcations, attractors, chaos. Not open to students with credit in PHYS 428 or [PHYS 4428](#). 3 lectures. Formerly MATH 416.

Course Learning Objectives	
1	Construct and use phase portraits in one and two dimensions to determine qualitative behavior of solutions to systems of nonlinear differential equations
2	Identify fixed points and determine their stability via linear stability analysis
3	Determine the existence and stability of periodic orbits using mathematical tools such as the Poincare-Bendixson Theorem and Poincare maps
4	Apply energy functions to investigate the global structure of phase portraits including the existence of periodic, homoclinic, and heteroclinic orbits
5	Identify and analyze elementary bifurcations on one- and two-dimensional manifolds
6	Define the key properties of chaotic systems and use these properties to determine if a system is exhibiting chaos
7	Apply nonlinear dynamic theory and techniques to analyze models drawn from physics, biology, engineering, and/or other fields
8	Use computational tools to analyze and simulate continuous dynamical systems in one and two dimensions

Expanded Course Content

Textbooks, materials, and/or other resources

To Be chosen by the instructor. Suggested texts include:

1. Strogatz, S., Nonlinear Dynamics and Chaos
2. Robinson R .C., An Introduction to Dynamical Systems
3. Hirsch, M. W., S. Smale, & R. L. Devaney, Differential Equations, Dynamical Systems, and an Introduction to Chaos

Week	Readings	Topics of Discussion
1	Strogatz, Sections 2.0 – 2.2	Introduction, flows on a line, fixed points and stability
2	Strogatz, Sections 2.3 – 2.6	Linear stability analysis, Uniqueness-Existence Theorem, a population growth model
3	Strogatz, Sections 2.7, 3.0 – 3.1	Potentials, introduction to bifurcations in one dimension (1D)
4	Strogatz, Sections 3.2 – 3.4	More bifurcations in 1D, laser model
5	Strogatz, Sections 3.6, 3.7	Imperfect bifurcations, an insect outbreak model
6	Strogatz, Chapter 5	Two-dimensional (2D) linear systems, a human interaction model
7	Strogatz, Sections 6.0 – 6.2	2D nonlinear systems and the phase plane, existence, uniqueness, and topological constraints of solutions
8	Strogatz, Sections 6.3, 6.4	Local linearization about fixed points, a competing-species model
9	Strogatz, Sections 6.5 – 6.7	Conservative and reversible systems, energy functions, the nonlinear pendulum

Week	Readings	Topics of Discussion
10	Strogatz, Sections 7.0 – 7.2	Introduction to limit cycles
11	Strogatz, Sections 7.3, 7.4	Poincare-Bendixson Theorem, Lienard systems
12	Strogatz, Sections 8.0 – 8.2	Basic bifurcations in 2D, the Hopf bifurcation
13	Strogatz, Sections 8.3, 8.4, 8.7	A chemical-reaction model, global bifurcations of limit cycles, Poincare maps
14	Strogatz, Sections 9.0 – 9.2	Introduction to chaotic systems, the Lorenz equations
15	Strogatz, Sections 9.3 – 9.4	Key properties of chaotic systems, strange attractors, the Lorenz map

Method of Assessment

The possible methods of assessment are written individual examinations, quizzes, in-class worksheets and presentations, projects, and homework. One or more midterms and a cumulative final exam are required.

Instructional Mode

The instructional mode for this class is face-to-face. This designation means that the essential academic standards of the course require in-class student engagement, and thus remote instruction is not appropriate.