

Course Outline for MATH 4265, Real Analysis II

MATH 4265 Real Analysis II (4 units)

Prerequisite: MATH 413 or [MATH 4264](#).

Further introduction to concepts and methods basic to real analysis. Concepts may include sequences and series of functions, differentiability of functions of several variables, Implicit and/or Inverse Function theorems, Riemann integration in several variables, and other advanced topics in real analysis. 4 lectures. Formerly offered as part of the MATH 412, 413, 414 series, students may not earn credit for both MATH 414 and [MATH 4265](#).

Course Learning Objectives	
1	Use definitions and basic theorems to determine convergence/divergence of sequences of functions.
2	Distinguish between pointwise and uniform convergence and describe and analyze properties of functions inherited by these different modes.
3	Use definitions and basic theorems to determine convergence/divergence of series of functions.
4	Compute radius and interval of convergence of Taylor series.
5	Distinguish between infinitely differentiable and real analytic functions by analyzing Taylor's remainder.
6	Prove properties of trigonometric, exponential, logarithmic, and other special functions.
7	Prove results about the point-set topological structure of n-dimensional real space.
8	Use definitions to determine differentiability of functions of several variables and prove basic theorems of differentiability familiar from calculus.
9	State and apply the Implicit Function Theorems.
10	Prove and apply Fubini's Theorem.
11	Apply the tools of analysis toward deeper study of advanced mathematical topics.

Expanded Course Content

Textbooks, materials, and/or other resources

1. Understanding Analysis, Second Edition, by Stephen Abbott
2. An Introduction to Analysis, Fourth Edition, by William R. Wade
3. A Glimpse of Fourier Analysis, by Dylan Retsek

Weeks 13 - 15 may include Riemann-Stieltjes integration, change of variables for multiple integrals, Baire category, Lebesgue measure on $\mathbb{R}$, Fourier analysis, functional analysis, Inverse Function Theorem, metric spaces or other capstone topics. A sample week 13 - 15 outline from Fourier analysis is given below.

Week	Readings	Topics of Discussion
1	Introduction to sequences of functions Abbott Sections 6.1, 6.2	Sequences of functions, pointwise and uniform convergence, inheritance of continuity and integrability, interchange of limits.
2	More on sequences and series of functions Abbott Sections 6.2 (continued), 6.3	Inheritance of differentiability, convergence of series in terms of sequence of partial sums, integrals and derivatives of series.
3	Advanced results on series of functions Abbott Sections 6.4, 6.7	The Cauchy criterion, Weierstrass M-Test, and Weierstrass approximation theorem.
4	Introduction to power series Abbott Section 6.5	Power series, Abel's theorem, term-by-term differentiation and integration.
5	Introduction to Taylor Series Abbott Section 6.6	Taylor series, Taylor's formula, remainder theorems, infinitely differentiable functions vs. real analytic functions.
6	Special Functions Wade Section 7.4	Exponential, trigonometric and logarithmic functions.

Week	Readings	Topics of Discussion
7	Stirling's formula Wade Section 12.6	Stirling's formula. Proof by way of the Wallis product vs. proof by way of the Gamma function, applications.
8	Introduction to n-dimensional Euclidean space Wade Sections 8.1 - 9.1	Triangle inequality, inner product, Cauchy-Schwarz inequality, parallelogram law, orthogonality, sequences, open, closed and compact sets.
9	Introduction to differentiability of functions on n-dimensional Euclidean space Wade Sections 11.1 - 11.3	Directional derivatives, definition of differentiability, implications between existence of partial derivatives and differentiability.
10	Advanced results on differentiability Wade Sections 11.4 - 11.6	Chain Rules, analogues to Taylor's Theorem and Mean Value Theorem, Implicit Function Theorem.
11	Differentiability of vector-valued functions Wade Sections 11.1 - 11.6	Frechet derivative, Chain Rule, Implicit Function Theorem for a System.
12	Introduction to Riemann integration on n-dimensional Euclidean space Wade Sections 12.1 - 12.3	Emphasis on 2-dimensional Euclidean space, the area problem, inner vs. outer area, Fubini's theorem for double integrals.
13	Introduction to Fourier series Retsek Section 1.1	Periodic and piecewise continuous functions on $\mathbb{R}$, Fourier coefficients and definition of the Fourier series of a function, initial convergence issues of the Fourier series.
14	Resolving convergence questions Retsek Section 1.2	Controlling the sequence of Fourier coefficients, square-summability of coefficients, the Dirichlet kernel.
15	A big result on convergence, the Fejer kernel Retsek Sections 1.3, 2.1	Piecewise smooth functions, convergence of the Fourier series for periodic, continuous, piecewise

Week	Readings	Topics of Discussion
		smooth functions, summation of p-series when $p=2$, Fejér kernel, Fejér summability.

Method of Assessment

The possible methods of assessment are written individual examinations, quizzes, in-class worksheets and presentations, projects, and homework. One or more midterms and a cumulative final exam are required.

Instructional Mode

The instructional mode for this class is face-to-face. This designation means that the essential academic standards of the course require in-class student engagement, and thus remote instruction is not appropriate.