

Course Outline for MATH 4264, Real Analysis I

MATH 4264 Real Analysis I (4 units)

Prerequisite: Graduate standing in Mathematics, MATH 306, or [MATH 3152](#).

Introduction to concepts and methods basic to real analysis. Concepts may include the real number system, sequences, continuity, differentiation and Riemann integration of functions of a single real variable. 4 lectures. Formerly offered as part of the MATH 412, 413, 414 series, students may not earn credit for both MATH 412 and [MATH 4264](#).

Course Learning Objectives	
1	Describe the construction and basic properties of the real number system.
2	Use definitions and basic theorems to determine convergence/divergence of sequences of real numbers.
3	Use definitions and basic theorems to determine convergence/divergence of series of real numbers.
4	Prove results about the point-set topological structure of the real numbers.
5	Use definitions to determine continuities and discontinuities of functions.
6	Distinguish between continuity and uniform continuity of a given function in terms of the topological properties of the domain.
7	Use definitions to determine differentiability of functions and prove basic theorems of differentiability familiar from calculus.
8	Prove and apply the Mean Value Theorem.
9	State and apply the definition of Riemann integrability to compute definite integrals and prove basic properties of the Riemann Integral.
10	Prove and apply the Fundamental Theorem of Calculus.
11	Classify Riemann integrability in terms of a function's set of discontinuities.

Expanded Course Content

Textbooks, materials, and/or other resources
Understanding Analysis, Second Edition, by Stephen Abbott

Week	Readings	Topics of Discussion
1	Introduction to the real number system Abbott Section 1.1	Natural numbers, integers, rational numbers, irrationality of the square root of 2.
2	The leap from the rational numbers to the real numbers Abbott Sections 1.2, 1.3	Dedekind cuts and the completeness property. Archimedean property, existence of square roots, nested interval property.
3	Introduction to sequences Abbott Sections 2.1, 2.2, 2.3	Definition of sequential convergence, algebraic properties of limits, monotonic sequence theorem.
4	Advanced results on sequences Abbott Sections 2.4, 2.5	Subsequences, Bolzano-Weierstrass theorem, Cauchy criterion.
5	Introduction to infinite series Abbott Sections 2.6, 2.7	Definition of convergence in terms of sequence of partial sums, algebraic properties of convergent series, geometric and telescoping sums.
6	Advanced results on infinite series Abbott Section 2.7 (continued)	Cauchy Condensation Test, p-Series, Ratio and Root Tests, conditional vs. absolute convergence, rearrangements.
7	Basic topology of the real numbers Abbott Sections 3.1, 3.2, 3.3	Open and closed sets, compactness, Bolzano-Weierstrass Theorem, the Cantor set.
8	Introduction to continuity Abbott Sections 4.1, 4.2, 4.3	Delta-epsilon definition of the limit of a function, definition of continuity, examples of Dirichlet and Thomae, algebraic properties of continuous functions.
9	Advanced results on continuity Abbott Sections 4.4, 4.5, 4.6	Uniform continuity, continuous functions on compact sets, Intermediate Value Theorem, sets of discontinuities.

Week	Readings	Topics of Discussion
10	Introduction to differentiability Abbott Sections 5.1, 5.2	Definition of derivative, continuity vs. differentiability, algebraic properties, power rule, product rule, quotient rule.
11	Advanced results on differentiability Abbott Sections 5.2 (continued), 5.3	Chain Rule, Mean Value Theorem, generalized Mean Value Theorem, L'Hospital's Rule.
12	More on differentiability Abbott Sections 5.3 (continued), 5.4	Darboux's Theorem, Existence of a Continuous Nowhere Differentiable Function.
13	Introduction to Riemann integrability Abbott Sections 7.1, 7.2	Definition of the Riemann integral, algebraic properties, integration of functions with finitely many discontinuities.
14	Fundamental Theorem of Calculus Abbott Sections 7.3, 7.4, 7.5	Statement and proof of the Fundamental Theorem of Calculus, consequences including integration by parts, and Change of Variables Theorem.
15	The Lebesgue Criterion for Integrability Abbott Section 7.6	Sets of measure zero, statement and proof of the Lebesgue Criterion for Integrability.

Method of Assessment

The possible methods of assessment are written individual examinations, quizzes, in-class worksheets and presentations, projects, and homework. One or more midterms and a cumulative final exam are required.

Instructional Mode

The instructional mode for this class is face-to-face. This designation means that the essential academic standards of the course require in-class student engagement, and thus remote instruction is not appropriate.