

Course Outline for MATH 4202, Abstract Algebra II

MATH 4202 Abstract Algebra II (3 units)

Prerequisite: MATH 482 or [MATH 4201](#).

Introduction to field theory. Factorization in polynomial rings, field extensions and Kronecker's theorem, splitting fields, finite fields, Galois groups and Galois field extensions, fundamental theorem of Galois theory. 3 lectures. Formerly offered as part of the MATH 481, 482, 483 series, students may not earn credit for both MATH 483 and [MATH 4202](#).

Course Learning Objectives	
1	Prove and apply theorems about divisibility and irreducibility of polynomials over rational numbers and finite fields.
2	Define algebraic elements and find their minimum polynomials over a field in special cases.
3	Construct a finite field extension containing a root of an irreducible polynomial.
4	Define and prove properties of finite field extensions of finite fields.
5	Prove crucial parts of the fundamental theorem of Galois theory.
6	Detail and prove properties about cyclotomic field extensions, including the determination of the Galois group.
7	Construct the tree of sub-extensions of the Galois extension defined by a pure polynomial of small degree over the rational numbers.
8	Apply Galois theory to another discipline, such as classical constructions with straightedge and compass, or solving general polynomials.

Expanded Course Content

Textbooks, materials, and/or other resources
1. E. Brussel, Straight Shot to Galois Theory
2. Dummit & Foote, Abstract Algebra
3. M. Artin, Algebra
4. J.P. Tignol, Galois' Theory of Algebraic Equations

Week	Readings	Topics of Discussion
1	Artin: 12.3, 12.4 D&F: 9.3	Field of fractions, divisibility in polynomial rings
2	Artin 12.3, 12.4 D&F: 9.4	Polynomial irreducibility over the rational numbers
3	Artin: 15.2, 15.3 D&F: 13.2	Field extensions, finite extensions, multiplicativity of degree
4	Artin: 15.6 D&F: 13.1, 13.2	Algebraic and transcendental elements, construction of a field containing a polynomial root, minimum polynomial
5	Artin: 16.3 D&F: 13.4	Splitting fields, existence theorem
6	Artin: 16.4 D&F: 13.1	Construction of splitting fields, uniqueness properties
7	Artin: 16.5, 16.6 D&F: 14.1	Galois group definition, interpretation as symmetry group of a polynomial
8	Artin: 15.7 D&F: 13.5	Finite fields: existence and uniqueness, finite fields as splitting fields
9	Artin: 15.7 D&F: 13.5	Finite field additive and multiplicative groups
10	Artin: 15.8, 16.5, 16.6, 16.7 D&F: 14.2	Galois theory: primitive element theorem, definition of Galois field extension, the fundamental theorem of Galois theory
11	Artin: 16.8 D&F: 14.7	Explicit computation of Galois correspondence, examples
12	Artin: 16.5 D&F: 14.1	Explicit computation of Galois correspondence, more examples, such as the general cubic

Week	Readings	Topics of Discussion
13	Artin: 15.5 D&F: 13.3	The Galois theory of cyclotomic extensions
14	D&F: 13.4, 13.6	Straightedge and compass construction of regular polygons
15	Artin: 16.12 D&F: 14.7	Instructor's choice (for example, the insolubility of the general quintic)

Method of Assessment

The possible methods of assessment are written individual examinations, quizzes, in-class worksheets and presentations, projects, and homework. One or more midterms and a cumulative final exam are required.

Instructional Mode

The instructional mode for this class is face-to-face. This designation means that the essential academic standards of the course require in-class student engagement, and thus remote instruction is not appropriate.