

Course Outline for MATH 4201, Abstract Algebra I

MATH 4201 Abstract Algebra I (4 units)

Prerequisite: One of the following: Graduate standing in Mathematics, MATH 306, MATH 341, [MATH 3111](#), or [MATH 3152](#).

Introduction to the study of groups and rings. Permutation groups, cyclic and dihedral groups, Lagrange's theorem, conjugation, quotient groups, isomorphism theorems, and automorphisms; basic ring types, ring homomorphisms, ideals, quotients, prime and maximal ideals, factorization in commutative rings. 4 lectures. Formerly offered as part of the MATH 481, 482, 483 series, students may not earn credit for both MATH 481 and [MATH 4201](#).

Course Learning Objectives	
1	Define examples of infinite and finite groups, and analyze their basic properties such as orders of elements and subgroup lattice.
2	Define how groups arise as sets of symmetries, such as symmetry groups of simple geometric objects.
3	Use the action of a group on a set to show how symmetry reflects subgroup structure.
4	Define and prove properties of the center and commutator subgroup of a finite group.
5	Define and prove properties about normal subgroups, quotient groups, and group homomorphisms.
6	Prove and apply the first, second, and third isomorphism theorems.
7	Define and prove properties of the automorphism group and outer automorphism group of a finite group.
8	Define and analyze the basic types of rings.
9	State and prove theorems about prime ideals and maximal ideals in commutative rings.
10	Prove theorems about the structure of Euclidean domains, principal ideal domains, and unique factorization domains.

Expanded Course Content

Textbooks, materials, and/or other resources
T. Judson, Abstract Algebra, Theory and Applications

Week	Readings	Topics of Discussion
1	Judson: 3.1, 3.2, 3.3	Groups as sets of symmetries of simple geometric objects; elementary properties and basic examples including Z , Z_n , Klein 4-group V , unit group $U(n)$, D_n , S_n , A_n ; subgroups
2	Judson: 4.1, 4.2	Cyclic groups, the order of an element, subgroups of cyclic groups, the order of a power; generators and relations for dihedral groups
3	Judson: 5.1	Symmetric and alternating groups, cycle notation, permutation matrices, how to conjugate in S_n
4	Judson: 5.2	Dihedral groups, the symmetries of regular polygons, the tetrahedron, and the cube; action of a group on a geometric set
5	Judson: 6.1, 6.2	Cosets and Lagrange's theorem; corollaries including Euler's theorem
6	Judson: 9.1, 9.2	Isomorphisms, direct products, Cayley's theorem
7	Judson: 10.1, 10.2	Normal subgroups, quotient groups, and the simplicity of A_n using generation by 3-cycles
8	Judson: 11.1, 11.2	Homomorphisms, first isomorphism theorem, subgroup correspondence theorem, second and third isomorphism theorems
9	Judson: 11.4, 14.1, 14.2	Automorphism group, inner automorphisms, conjugation in S_n , conjugacy classes, class equation, the simplicity of A_5 by class equation
10	Judson: 16.1, 16.2	Definition of ring and ideal, and examples, including simple, division, commutative, domain, UFD (unique factorization domain), PID (principal ideal domain), ED (Euclidean domain), field
11	Judson: 16.1, 16.2 (continued)	Homomorphisms and quotient rings
12	Judson: 16.3	First isomorphism theorem, ideal correspondence theorem, direct product

Week	Readings	Topics of Discussion
13	Judson: 16.4	Commutative rings: maximal ideals, prime ideals, criteria, nil radical, prime spectrum
14	Judson: 18.1, 18.2	Euclidean domains, ED implies PID, examples of ED's
15	Judson: 18.1, 18.2 (continued)	Factorization in integral domains, PID implies UFD

Method of Assessment

The possible methods of assessment are written individual examinations, quizzes, in-class worksheets and presentations, projects, and homework. One or more midterms and a cumulative final exam are required.

Instructional Mode

The instructional mode for this class is face-to-face. This designation means that the essential academic standards of the course require in-class student engagement, and thus remote instruction is not appropriate.