

Course Outline for MATH 3351, Differential Equations and Boundary Value Problems

MATH 3351 Differential Equations and Boundary Value Problems (3 units)

Prerequisite: Junior standing or one of the following: Biomedical Engineering major, Mathematics major, or Mechanical Engineering major; completion of GE Area 1 with grades of C- or better; and completion of GE Area 2 with a grade of C- or better; and one of the following: MATH 242, MATH 244C, [MATH 2341](#), or [MATH 2343](#); and one of the following: MATH 241, DATA/[MATH 1265](#), or [MATH 2263](#).

Linear theory and methods applied to the solution of differential equations. Series solutions and Frobenius theory. Bases and orthogonality in function spaces, Gram-Schmidt orthogonalization. Regular and singular Sturm-Liouville theory. Fourier series and transforms. Special functions of applied mathematics. 3 lectures. Fulfills GE Areas Upper-Division 2 or Upper-Division 5. Formerly MATH 344.

Course Learning Objectives	
1	Use Taylor series and Frobenius theory to find approximate solutions to differential equations near ordinary and regular singular points.
2	Define and prove basic properties of function spaces with inner products including orthogonality and convergence-in-the-mean.
3	Use complete orthogonal sets to approximate functions in function spaces.
4	Define Fourier series and their basic properties.
5	State and prove key components of regular and singular Sturm-Liouville theory.
6	Apply Sturm-Liouville theory to find eigenfunction expansions for solutions to two-point boundary value problems.
7	Use the method of separation of variables and Sturm-Liouville theory to solve introductory partial differential equations such as the linear heat equation in one spatial dimension.
8	Define and use Fourier transforms to solve boundary value problems on unbounded domains.
9	Define and determine basic properties of key special functions such as Legendre polynomials and Bessel functions.

Expanded Course Content

Textbooks, materials, and/or other resources

- A. Instructors can choose from a variety of texts including one of the followings:
1. Boyce, DiPrima, and Meade, Differential Equations and Boundary value Problems
 2. Churchill and Brown, Fourier Series and Boundary Value Problems
 3. Nagle, Saff, and Snider, Differential Equations and Boundary Value Problems
- B. Most texts will require some supplementary material

Week	Readings	Topics of Discussion
1	Boyce, et al. Sec 5.1, 5.2, 5.3	Review local Taylor series representations and Taylor polynomial approximations for analytic functions. Classification of ordinary points and singular points of an ordinary differential equation. Series expansions at ordinary points.
2	Boyce, et al. Sec 5.4, 5.5	Cauchy-Euler equations. Series expansions at regular singular points. Introduction to the Method of Frobenius.
3	Boyce, et al. Sec 5.6, 5.7	Method of Frobenius for repeated roots and roots differing by integers. Bessel's Equation.
4	Supplementary material	Function spaces. Inner products and orthogonality for function spaces. Complete, orthogonal sets.
5	Supplementary material	Function approximation. Convergence in the mean. Gram-Schmidt orthogonalization. Legendre polynomials as a complete, orthogonal set.
6	Boyce, et al. Sec 10.1, 10.2	Two-point boundary value problems. Fourier series representations of functions as an expansion over a complete, orthogonal set.
7	Boyce, et al. Sec 10.3, 10.4	Convergence of Fourier series. Gibbs phenomena. Fourier Sine and Cosine series.
8	Boyce, et al. Sec 10.5, 10.6	Basic linear models of heat conduction. Separation of variables. Connection of Fourier series to partial differential equations.

Week	Readings	Topics of Discussion
9	Boyce, et al. Sec 11.2 Supplementary material	Lagrange's Identity. Adjoint operators and boundary conditions. Self-adjoint boundary value problems. Regular Sturm-Liouville Eigenvalue Problems. Connection to Fourier Sine and Cosine series.
10	Boyce, et al. Sec 11.2	Proof of selected components of regular Sturm-Liouville theory. Extension to periodic boundary conditions. Connection to Fourier series.
11	Boyce, et al. Sec 11.4, 11.5	Singular Sturm-Liouville eigenvalue problems. Bessel Functions and Legendre polynomials revisited as bases for orthogonal function expansions.
12	Boyce, et al. Sec 11.3, 11.6 Supplementary material	Eigenfunction expansions. Nonhomogeneous problems. Convergence of eigenfunction expansions.
13	Supplementary material	Eigenfunction expansions for simple linear PDEs such as heat flow in a rod.
14	Supplementary material	Unbounded domains and the Fourier transform. The connection between Fourier transforms and series.
15	Supplementary material	Applying the Fourier Transform to solve simple linear PDEs on unbounded domains such as heat flow in an infinitely long rod.

Method of Assessment

The possible methods of assessment are written individual examinations, quizzes, in-class worksheets or presentations, projects, and homework. One or more midterms and a cumulative final exam are required.

Instructional Mode

The instructional mode for this class is face-to-face. This designation means that the essential academic standards of the course require in-class student engagement, and thus remote instruction is not appropriate.