

Course Outline for MATH 3152, Advanced Linear Algebra

MATH 3152 Advanced Linear Algebra (4 units)

Prerequisite: One of the following: MATH 206, MATH 244, [MATH 1151](#), or [MATH 2341](#); and MATH 248 or [MATH 2031](#) with a grade of C- or better.

Rigorous development of real and complex vector spaces. May include subspaces, bases, direct sums, linear maps, eigenvalues, eigenvectors, diagonalization, inner product spaces, self-adjoint and normal operators, spectral theorems, nilpotent operators, characteristic and minimal polynomials, Cayley-Hamilton Theorem. 4 lectures. Formerly MATH 306.

Course Learning Objectives	
1	Use definitions and methods of linear algebra to prove results about vector spaces and their properties.
2	Prove results concerning subspaces, spanning sets, linear independence, bases, and dimension.
3	Prove results concerning linear maps and their properties.
4	Apply the Gram-Schmidt procedure and use inner product computations to solve minimization problems in function spaces.
5	Prove consequences of the complex and real spectral theorems.
6	Prove basic results concerning generalized, nilpotent operators, and the decomposition of an operator.

Expanded Course Content

Textbooks, materials, and/or other resources

To be chosen by the instructor. Suggested textbooks include:

1. Axler, Sheldon, Linear Algebra Done Right
2. Friedberg, Stephen, et al., Linear Algebra
3. Lewis, David, Matrix Theory
4. Meyer, Carl, Matrix Analysis and Applied Linear Algebra

The Expanded Course Content is based on Axler, Sheldon, Linear Algebra Done Right.

Week	Readings	Topics of Discussion
1	Sections 1.A, 1.B, 1.C	$\mathbb{R}^n$ and $\mathbb{C}^n$, Definition of a Vector Space, Subspaces, Sums of Subspaces
2	Sections 1.C, 2.A	Subspaces, Sums of Subspaces, Span and Linear Independence
3	Section 2.B	Bases, Dimension
4	Sections 3.A, 3.B	The Vector Space of Linear Maps, Null Spaces and Ranges, Fundamental Theorem of Linear Maps
5	Sections 3.C, 3.D	Matrices, Representation of Linear Maps, Invertibility and Isomorphisms
6	Sections 3.D, 4, 5.A	Invertibility and Isomorphisms, Polynomials, Factorization Theorems, Invariant Subspaces, Eigenvalues and Eigenvectors
7	5.A, 5.B	Invariant Subspaces, Eigenvalues and Eigenvectors, Upper Triangular Matrices, Polynomial Operators and the Existence of Eigenvalues
8	Sections 5.C, 6.A	Eigenspaces and Diagonal Matrices, Inner Products and Norms
9	Sections 6.A, 6.B	Inner Products and Norms, Orthonormal Bases, Gram-Schmidt Procedure
10	Sections 6.C, 6.B	Orthogonal Complements and Minimization Problems, Linear Functionals on Inner Product Spaces, Riez Representation Theorem
11	Section 7.A	Self-Adjoint and Normal Operators
12	Section 7.B	The Spectral Theorems
13	Sections 7.C, 7.D	Positive Operators and Isometries, Polar Decomposition and the Singular Value Decomposition Theorem

Week	Readings	Topics of Discussion
14	Sections 8.A, 8.B	Generalized Eigenvectors and Nilpotent Operators, Decomposition of an Operator, Multiplicity of an Eigenvalue
15	Sections 8.B, 8.C, 8.D	Decomposition of an Operator, Block Diagonal Matrices, Characteristic and Minimal Polynomials, Jordan Form

Method of Assessment

The possible methods of assessment are written individual examinations, quizzes, in-class worksheets or presentations, projects, and homework. One or more midterms and a cumulative final exam are required.

Instructional Mode

The instructional mode for this class is face-to-face. This designation means that the essential academic standards of the course require in-class student engagement, and thus remote instruction is not appropriate.