Mathematics Colloquium

The Stack of Triangles up to Similarity

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Friday, October 3, 2025 11:10 am – 12 pm Building 38, Room 219

Abstract

In 1880 Schubert constructed what is now viewed as a smooth compactification of the space of labeled, oriented (possibly-degenerate) triangles in the plane, as part of his enumerative calculus. In this talk I will discuss our construction of the space of similarity classes of labeled, oriented triangles, which should be a quotient of Schubert's space under the group of rigid motions and scaling. Such a "shape space" of triangles (or moduli space of triangle classes) should be compact and smooth, and have a universal property with respect to continuous families of similarity classes. Our space is the connected sum of three (real) projective planes, also known as Dyck's surface. It incorporates other smooth parameterizations that have appeared in the literature, but that had failed to be universal: the real 2-sphere, in work of Kendall, Portnoy, Behrend, Montgomery, and others; and the 2-torus, in Brussel-Goertz. This work is joint with Cal Poly students Goertz, Lyle, and Guptill, Brussel's 2023 Frost summer research cohort.

About the speaker: Eric Brussel has been a member of the Cal Poly Mathematics professoriate for 13 years. He received his Ph.D. in 1993 at UCLA under the supervision of algebraist Murray Schacher, based on a construction of certain distinguished division algebras over function fields of curves over \mathbb{Q} . Most of his research has concerned the Brauer group over arithmetic function fields, often over p-adic fields.