Abstract

The Kepler-Heisenberg problem is that of determining the motion of a planet around a sun in the Heisenberg group, thought of as a three-dimensional sub-Riemannian manifold. The sub-Riemannian Hamiltonian provides the kinetic energy, and the gravitational potential is given by the fundamental solution to the sub-Laplacian. The system was recently shown to be non-integrable, but there is a fundamental integrable subsystem consisting of orbits with zero energy. This subsystem contains a wealth of beautiful dynamics, including closed orbits of every rational rotation number. Here, we demonstrate that all zero energy orbits are actually self-similar. Technicalities will be minimal and pictures will be plentiful. We conclude with some open problems.

About the speaker: Dr. Shanbrom received his PhD from UCSC under Richard Montgomery. He is currently Professor and Vice Chair in the Department of Mathematics and Statistics at Sacramento State University. His research interests include geometry, dynamics, and singularities.