

Mathematics Colloquium

The Discovery of the Three-Dimensional Sphere

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4:10 am – 5 pm
Building 180, Room 114

Abstract

Every Euclidean n -space contains a unit $(n-1)$ -sphere, which is the set of points at distance 1 from the origin. For example, 3-space has a 2-sphere, which is the familiar surface of a round ball. But of all the spheres in all of the Euclidean spaces, only in 2-space and in 4-space do their points correspond bijectively to the space's rotational symmetry group. This gives them an important role in the description of physical systems. From its home up high in 4-space, the 3-sphere seems to preside over the rotational dynamics of our 3-space. From there it also plays an indispensable role in the description of the quantum spin behavior of elementary particles, in a way that is in some ways mysterious. Albert Einstein, in the aftermath of his discovery that physical space is actually curved, not flat, hypothesized that our space is topologically a 3-sphere, an idea that would have resolved the apparent paradox between 3-space's evident boundedness and our inability to conceive of a boundary.

We will describe the 3-sphere as best we can given our limited ability to visualize four dimensions, and tell the story of how its existence as an absolutely fundamental behind-the-scenes presence was surmised from one person's close examination of the rotations of a basketball. We will discuss the theme of implausibly rich mathematics being "discovered" in this way, even though, as a conceptual narrative, these things have no conceivable existence outside of a mind. Finally, we will show how our model is beautifully described in Dante's *Paradiso* and portrayed by the artist Gustave Dore, an observation pointed out by the physicist Carlo Rovelli.

About the speaker: Eric Brussel got his Ph.D. at UCLA under the supervision of Murray Schacher. He is interested in a wide variety of subjects related to classifying and understanding the structure of finite-dimensional division algebras, especially those that exist over the function fields of p -adic curves.