

2018 Summer Research Project - Eric Brussel

Project Description: Topics in algebraic number theory and algebraic geometry

The 3-sphere is the set of points of 4-dimensional Euclidean space of distance one from the origin. The 3-sphere is a group, in fact it is an algebraic group, because it is also an algebraic variety. We wish to investigate its subgroup of rational points, which are points whose coordinates are in the rational numbers. In particular we would like to look at the arithmetic complexity of rational points, and how they are distributed in the 3-sphere. Areas of interest for this project include some beginning algebraic geometry and algebraic number theory, including Diophantine approximation, which is about approximating arbitrary points with rational points, and the theory of continued fractions, which gives a means of finding good rational approximations. In the end, we'd like to draw a picture of low-complexity points of the 3-sphere. Prerequisites include the first two quarters of abstract algebra.