

# Frost Summer 2018 Support Proposal

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**Abstract:** Given a set  $S$  of functions we often ask

*If  $f$  and  $g$  are members of  $S$  and both enjoy property  $X$ ,  
does  $f \circ g$  also enjoy property  $X$ ?*

When  $S$  is the set of functions from  $\mathbb{R}$  to  $\mathbb{R}$  and property  $X$  is continuity or differentiability, we answer this question affirmatively in calculus class.

In this project we shall consider the Hardy space  $H^2(\mathbb{D})$  of analytic functions on the open unit disk  $\mathbb{D}$  in the complex plane whose Maclaurin coefficients are square-summable. Given analytic  $\varphi : \mathbb{D} \rightarrow \mathbb{D}$ , the composition operator  $C_\varphi$  on  $H^2(\mathbb{D})$  is then defined by  $C_\varphi f = f \circ \varphi$ .

One program of research is to understand the operator-theoretic properties of  $C_\varphi$  in terms of the geometric-function-theoretic properties of the map  $\varphi$ . We will continue a few advances made along these lines during Frost research in the summer of 2017 and also pursue a closely related undergraduate research project begun this school year by math major Jakob Loren. This project is designed for undergraduates who have had calculus, methods of proof, linear algebra and some elementary complex analysis.