Course Outline for MATH 5371, Methods of Applied Mathematics

MATH 5371 Methods of Applied Mathematics (3 units)
Prerequisite: One of the following: Graduate standing in Mathematics, MATH 410, MATH 412, MATH 3301, or MATH 4264.

Advanced mathematical methods of analysis in science and engineering, integrated with modeling of physical phenomena. Asymptotics, perturbation methods, and boundary layer analysis. Additional content to be drawn from WKB theory, multi-scale analysis, and the calculus of variations. 3 lectures. Formerly MATH 521.

	Course Learning Objectives
1	Compare and contrast asymptotic methods for finding approximate solutions to linear and nonlinear ordinary differential equations
2	Define and prove results related to the concepts of order relations, asymptotic sequences, and asymptotic expansions
3	Use asymptotic expansions and the method of dominant balance to find approximate solutions to linear ordinary differential equations near irregular singular points
4	Define and distinguish regular and singular perturbation theories
5	Use regular perturbation theory to find approximate solutions to ordinary differential equations and identify the limitations of the resulting solutions
6	Use the method of dominant balance and asymptotic matching to find uniform leading and higher-order expansions for boundary-layer problems
7	Use advanced techniques such as WKB theory, multi-scale analysis, or the calculus of variations to find technique-appropriate solutions for various regular and singular perturbation or variational problems

Expanded Course Content

Textbooks, materials, and/or other resources
1. Bender and Orszag, Advanced Mathematical Methods for Scientists and Engineers I, Springer 2. Dacorogna, Introduction to the Calculus of Variations, 2nd edition, World Scientific Publishing

Week	Readings	Topics of Discussion
1	B&O Sections 3.4, 3.5	Initial examples, order relations, asymptotic sequences and expansions
2	B&O Sections 3.5, 3.7	Irregular singular points, asymptotic power series, method of dominant balance
3	B&O Sections 3.7, 3.8	Rigorous treatment of the properties of general asymptotic expansions
4	B&O Sections 7.1	Regular and singular perturbations
5	B&O Sections 7.2	Regular and singular perturbations, breakdown of regular perturbation techniques in ordinary differential equations
6	B&O Sections 7.4	Asymptotic matching, introduction of boundary layer problems
7	B&O Sections 9.1	Boundary layer theory, linear and nonlinear examples
8	B&O Sections 9.2, 9.4	Structure of boundary layers, uniform expansions
9	B&O Sections 9.3, 9.5	Boundary layer theory, higher order expansions, examples of limitations and failures
10	B&O Sections 10.1, 10.2	WKB Theory, conditions for validity
11	B&O Sections 10.2, 10.4	WKB Theory, one-turning-point problem
12	B&O Sections 11.1, 11.2	Multi-scale analysis, resonance, secular growth
13	B&O Sections 11.3, 11.4	Multi-scale analysis, elementary Floquet theory and the Mathieu equation
14	Dacorogna Sections 2.1, 2.2, 2.3	Calculus of variations, Euler-Lagrange equation
15	Dacorogna Sections 2.4, 2.5	Hamiltonian formulation of mechanics, Hamilton-Jacobi equation

Method of Assessment

The possible methods of assessment are written individual examinations, quizzes, in-class worksheets and presentations, projects, and homework. One or more midterms and a cumulative final exam are required.

Instructional Mode

The instructional mode for this class is face-to-face. This designation means that the essential academic standards of the course require in-class student engagement, and thus remote instruction is not appropriate.