Course Outline for MATH 5266, Real Analysis

MATH 5266 Real Analysis (3 units)
Prerequisite: Completion of the Graduate Written Examination in Analysis.

Introduction to measure and integration, Lebesgue measure, Lebesgue integration, convergence theorems, Fubini's Theorem and the Radon-Nikodym Theorem. 3 lectures. Formerly MATH 550.

	Course Learning Objectives
1	Apply the definition of measure in the abstract setting of sigma-algebras and prove the basic properties of measures.
2	Construct measures by way of outer measures and Caratheodory's theorem.
3	Construct measures on R, specifically Lebesgue-Stieltjes measures.
4	Use simple functions to define integration against a measure and prove basic properties of integrals.
5	Prove and apply Lebesgue Monotone Convergence Theorem, Fatou's Lemma and Lebesgue Dominated Convergence Theorem to limiting problems involving integrals.
6	Compare and contrast Lebesgue Integration to Riemann Integration.
7	Compare and contrast various modes of convergence for sequences of measurable functions.
8	Construct and integrate with respect to product measures.
9	Decompose signed measures into mutually singular parts.
10	State and apply the Radon-Nikodym Theorem.
11	Prove and apply Fubini's Theorem.

Expanded Course Content

Textbooks, materials, and/or other resources
Real Analysis: Modern Techniques and Their Applications, Second Edition, by Gerald B. Folland

Week	Readings	Topics of Discussion
1	Introduction Folland Sections 1.1 - 1.2	Sigma-algebras, measurable spaces, existence of non-measurable sets.
2	Measures Folland Sections 1.2 - 1.3	Definition of measure, basic properties, null sets, completeness.
3	Outer Measures Folland Section 1.4 - 1.5	Outer measures, Caratheodory's Theorem, specification to "length" on the real line.
4	Lebesgue-Stieltjes Measures Folland Section 1.5	Construction of Lebesgue-Stieltjes measures, properties of Lebesgue measure, topology vs. measure, Cantor set.
5	Measurable Functions Folland Section 2.1	Measurable functions, preservation of measurability under algebraic and limiting processes.
6	Simple Functions Folland Section 2.2	Definition of simple functions, density within measurable functions, completion of a measure space.
7	Integration in L+ Folland Section 2.2	Integration of simple functions, extension to L+ via supremum, basic properties of the integral.
8	Convergence Theorems Folland Section 2.3	Lebesgue Monotone Convergence Theorem, Fatou's Lemma, integration of complex functions, Lebesgue Dominated Convergence Theorem.
9	Consequences Folland Section 2.3 - 2.4	Consequences of the convergence theorems, differentiation under the integral, Riemann integration versus Lebesgue integration, introduction to modes of convergence.
10	Modes of Convergence Folland Section 2.4	Convergence pointwise a.e., pointwise, uniformly, in L1 and in measure, Egoroff's Theorem.
11	Product Measures Folland Section 2.5	Definition of product measures, measurable functions, integration against product measures, statement of Fubini's Theorem.
12	Fubini's Theorem Folland Section 2.5	Proof of Fubini's Theorem, consequences and examples.

Week	Readings	Topics of Discussion
13	Signed Measures Folland Section 3.1	Definition of signed measure, Hahn Decomposition Theorem, Jordan Decomposition Theorem, mutual singularity and absolute continuity.
14	Complex Measures Folland Section 3.2	Complex measures, Radon-Nikodym Theorem, consequences.
15	Differentiation of Measures Folland Section 3.3	Differentiation with respect to Lebesgue measure, local integrability, Maximal Theorem.

Method of Assessment

The possible methods of assessment are written individual examinations, quizzes, in-class worksheets and presentations, projects, and homework. One or more midterms and a cumulative final exam are required.

Instructional Mode

The instructional mode for this class is face-to-face. This designation means that the essential academic standards of the course require in-class student engagement, and thus remote instruction is not appropriate.