Course Outline for MATH 5203, Linear Algebra and Module Theory

MATH 5203 Linear Algebra and Module Theory (3 units)
Prerequisite: Completion of the Graduate Written Examination in Algebra.

Modules, basic theorems, products and coproducts via universal properties, free modules, and modules over a PID. Applications to linear operators, bilinear forms, corresponding matrix groups; spectral theorems for Hermitian, normal, and unitary operators. Singular values, applications, and introduction to categories. 3 lectures.

Course Learning Objectives	
1	Define the categorical product and coproduct.
2	Use universal properties to prove facts about universal constructions.
3	Prove a submodule of a finitely generated module over a PID is finitely generated.
4	Calculate the invariant factors of a linear transformation in a finite-dimensional vector space and use the transformation to determine the Jordan canonical forms for small matrices.
5	Determine whether matrices are diagonalizable or similar using the fundamental theorem of finitely generated modules over a PID.
6	Define and give examples of bilinear spaces.
7	Construct an orthogonal basis for a symmetric bilinear space.
8	Prove and apply the spectral theorems for Hermitian, normal, and unitary operators.
9	Characterize with proof the linear groups that preserve the dot product and Hermitian product.
10	Compute the singular values and singular value decomposition of a matrix.

Expanded Course Content

Textbooks, materials, and/or other resources
Instructors can choose from a variety of textbooks and/or additional readings; often more than one text will be required. (There is free access to digital editions for many appropriate texts via the CP library). Possible textbooks include:

Textbooks, materials, and/or other resources
1. Dummit & Foote, Abstract Algebra, 3rd Edition 2. M. Artin, Algebra, 2nd Edition 3. S. Roman, Advanced Linear Algebra, 3rd Edition The sample course outline below is based on Dummit & Foote, Artin, and Roman.

Week	Readings	Topics of Discussion
1	D&F A.II.1, 1.3 (Review), 1.7	Introduction to categories, categorical products and coproducts. Review of symmetric groups and group actions.
2	D&F 10.1, 10.2	Modules over a commutative ring: definitions and examples; quotients and morphisms.
3	D&F A.II.2, 10.3	Categorical products and coproducts; module products and coproducts.
4	D&F 10.3 (continued), 12.1	Free modules, submodules of a free module over a PID.
5	D&F 12.1 (continued)	Finitely generated modules over a PID.
6	D&F 12.2	Applications to linear transformations over a field: Rational and Jordan canonical forms.
7	D&F 12.3	Applications to linear transformations (continued): Rational and Jordan canonical forms.
8	Artin 8.1, 8.2, 8.3	Symmetric and Hermitian bilinear forms, associated matrix, and equivalence.
9	Artin 8.4	Orthogonality with respect to a bilinear form, orthogonal decomposition, orthogonal basis.
10	Artin 8.4 (continued), 8.5	Projection formula and corollaries.
11	Artin 8.6	Adjoint operator, normal, Hermitian, and unitary matrices, spectral theorems.
12	Artin 8.6 (continued), 8.7	Spectral theorems, applications to quadratic forms and conics.

Week	Readings	Topics of Discussion
13	Roman, Chapter 17	Singular values and the SVD
14	To be determined by the instructor	Applications (instructor's choice)
15	To be determined by the instructor	Applications (instructor's choice)

Method of Assessment

The possible methods of assessment are written individual examinations, quizzes, in-class worksheets and presentations, projects, and homework. One or more midterms and a cumulative final exam are required.

Instructional Mode

The instructional mode for this class is face-to-face. This designation means that the essential academic standards of the course require in-class student engagement, and thus remote instruction is not appropriate.