MATH 502 Numerical Methods in Applied Mathematics

1. Catalog Description

MATH 502 Numerical Methods in Applied Mathematics **4 units**

Prerequisite: MATH 344 or AERO 300, an introductory college-level programming course, and graduate standing.

Introduction to advanced numerical analysis. Numerical techniques for solving ordinary and partial differential equations, error analysis, stability, methods for linear systems. Not open to students in math major or master's degree program in mathematics. 4 lectures.

2. Required Background or Experience

MATH 344 or AERO 300, an introductory college-level programming course, and graduate standing.

3. Learning Objectives

The student should:

a. Assess the appropriateness of a given numerical scheme for the solution of an ordinary or partial differential equation by calculating the accuracy and determining the stability of the scheme.
b. Apply standard numerical techniques to solve ordinary and partial differential equations.

4. Text and References

The text is to be chosen by the instructor. Suggested texts include:

- Randall LeVeque, Finite Difference Methods for Ordinary and Partial Differential Equations: Steady-State and Time-Dependent Problems, SIAM, 2007.
- Gilbert Strang, Computational Science and Engineering, 2007.
- Gene Golub and Charles van Loan, Matrix Computations, 3rd ed, 1996.
- Trefethen and Bau, Numerical Linear Algebra, SIAM 1997.

5. Minimum Student Materials

Paper, pencils, notebook, and access to computing equipment.

6. Minimum University Facilities

Classroom with ample chalkboard space and computer lab.

7. Expanded Course Content

Below is one possible week-by-week outline:

Week 1
Errors, big-oh, little-oh notation. Norms of vectors, functions, matrices. Finite difference approximations. Truncation errors. Deriving finite difference approximations. Application to boundary value problems.

Week 2
Local truncation error. Stability of a numerical scheme. Consistency and convergence. Relationship between discrete matrix problem and Green's functions. Compatibility conditions for Neumann boundary value problem.

Week 3
Discretization of elliptic partial differential equations with the 5-point stencil. Iterative methods for solving sparse linear systems. Jacobi and Gauss-Seidel. Method of steepest descent.

Week 4
Method of steepest descent. Conjugate gradient method. Multigrid methods.

Week 5
Initial value problems. Duhamel's principle. Lipschitz continuity. Existence and uniqueness of solutions. Local truncation error and one-step errors. Runge-Kutta methods. Linear multistep methods.

Week 6
Zero-stability and convergence of initial value problems. Absolute stability for ordinary differential equations.

Week 7
Stability regions for one-step and linear multistep methods. Stiff ordinary differential equations. A-stability and L-stability.

Week 8
Diffusion equations, heat equation. Local truncation errors. Method of lines. Stiffness of the heat equation. Crank-Nicolson method.

Week 9
Von Neumann stability analysis of elliptic PDEs. Hyperbolic PDEs: advection equation. Method of lines for hyperbolic PDEs. Lax-Friedrichs and Lax-Wendroff methods.

Week 10
Upwind methods. Courant-Friedrichs-Lewy (CFL) condition. Hyperbolic systems. Initial boundary value problems. PDEs of mixed type.

8. Methods of Assessment

Exams, homework, and possibly student presentations.