MATH 501 Analytic Methods in Applied Mathematics

1. Catalog Description

   MATH 501 Analytic Methods in Applied Mathematics **4 units**

   Prerequisite: MATH 344 or AERO 300, and graduate standing.

   Introduction to advanced methods of mathematics useful in the analysis of engineering problems. Selected topics in perturbation theory, optimization and Fourier analysis. Not open to students in math major or master's degree program in mathematics. 4 lectures.

2. Required Background or Experience

   MATH 344 or AERO 300, and graduate standing.

3. Learning Objectives

   The student should:
   a. Apply standard techniques in perturbation theory to solve ordinary differential equations.
   b. Compute solutions to optimization problems using the Calculus of Variations and adjoint methods.
   c. Analyze time series and differential equations using Fourier series and the Fourier transform.

4. Text and References

   The text is chosen by the instructor. Suggested texts include:

   - E. J. Hinch, Perturbation Methods. Cambridge University Press.
   - Gilbert Strang, Computational Science and Engineering.

5. Minimum Student Materials

   Paper, pencils, computing equipment, and notebook.

6. Minimum University Facilities

   Classroom with ample chalkboard space for class use and computer lab/computing facilities.

7. Content and Method

   Week 1
   Asymptotic series, big-oh, little-oh notation. Regular and singular perturbation solutions of algebraic equations. Singular perturbation solutions of ordinary differential equations. Dominant balance to find thickness of boundary layer.

Week 2
Matched asymptotic expansions and boundary-layer analysis. Outer solution, inner (boundary layer) solution, leading-order matching principle, matching by intermediate variable. Construction of uniformly valid expansion.

Week 3
WKB analysis of ordinary differential equations. Initial value problems. Boundary value problems, eigenvalue problems. Turning-point problems.

Week 4
Multiple-scale analysis. Secular terms and secular growth. Duffing's equation. Van der Pol oscillator.

Week 5
Intro to optimization. Unconstrained optimization: normal equations. Lagrange multipliers in multiple dimensions. Dual problem. Adjoint of a matrix operator.

Week 6
Calculus of Variations. Euler-Lagrange equations. The variational derivative. Principle of least action and Hamilton's principle.

Week 7
Adjoint methods. The adjoint of a differential operator. Examples of adjoint methods in engineering: shape optimization, data assimilation.

Week 8
Review of Fourier series, computation of Fourier coefficients. Relationship between speed of convergence of Fourier series and smoothness of function. Gibbs phenomenon.

Week 9
Fourier integrals and transform. Discrete Fourier transform and FFT. Signal analysis by Fourier decomposition.

Week 10
Relationship of physical space decay to frequency space decay: Heisenberg uncertainty principle. Solution of differential equations by Fourier transform, relationship to Green's functions.

8. <u>Methods of Assessment</u>

Exams, homework, and possibly student presentations.