MATH 453 Numerical Optimization

1. Catalog Description

MATH 453 Numerical Optimization **4 units**

Prerequisite: MATH 306 and MATH 451.

Algorithms for solving optimization problems that cannot be solved analytically. Descent algorithms including exact and practical line-searches, steepest descent method, and Newton and quasi-Newton methods for unconstrained minimization. Optimality conditions for constrained optimization, linear programming. Projection and Lagrangian methods, and interior point methods for constrained minimization. 4 lectures.

2. Required Background or Experience

Prerequisite: MATH 306 and MATH 451, or consent of instructor.

In order to understand numerical optimization, students must have an understanding of linear algebra from MATH 306, and a working knowledge of the techniques of numerical analysis from MATH 451.

3. Learning Objectives

Upon completion of this course students should:

a. Be familiar with the mathematical foundations and practical aspects of numerical optimization.
b. Have the modeling skills to formulate appropriate optimization problems.
c. Be able to implement algorithms using existing optimization software.

4. Text and References

Text to be chosen by the instructor. Suggested texts include:

- Chang, Edwin & Stanislaw, Zak. An Introduction to Optimization.

5. Minimum Student Materials

Access to computing equipment to allow implementation of numerical procedures.

6. Minimum University Facilities

Classroom with ample chalkboard space for class use and appropriate computing facilities.

7. Content

Topics

a. Review of Calculus and linear algebra
   1. Eigenvalues and eigenvectors
   2. Positive semidefinite matrices and quadratic forms
   3. Gradient, Jacobian, Hessian
   4. Taylor's formula for functions of several variables

b. Optimality conditions for optimization
   1. Constraint sets, feasible directions, first order necessary conditions
   2. Second order necessary and sufficient conditions

c. Algorithms for unconstrained optimization
   1. Golden section method
   2. Steepest gradient algorithm
   3. Conjugate gradient algorithm
   4. Newton and quasi-Newton algorithms

d. Linear programming
   1. geometric view (convex sets and extreme points)
   2. algebraic view (simplex method)
   3. duality
   4. saddle points, complementary slackness

e. Constrained optimization I (equality constraints)
   1. Lagrange multipliers:
      algebra and geometry of the Lagrangian
   2. Lagrange multipliers:
      as dual variables; sensitivity
   3. algorithms: Lagrangian method, projection gradient methods
      algebra and geometry of the Lagrangian

f. Constrained optimization II (inequality constraint)
   1. Kuhn-Tucker theory: algebra and geometry of Kuhn-Tucker vector
   2. Duality and sensitivity:
   3. algorithms: Interior point method, penalty methods

8. Methods of Assessment

The primary methods of assessment are: essay examinations, computing projects, quizzes and homework. Typically, there will be one or more hour-long examinations during the quarter, and a required comprehensive final examination. Students are required to show their work and are graded not only on the correctness of their answers, but also on their understanding of the concepts and techniques.