MATH 416 Differential Equations II

1. <u>Catalog Description</u>

   MATH 416 Differential Equations II   **4 units**

   Prerequisite: <u>MATH 206</u> and <u>MATH 242</u>, or <u>MATH 241</u> and <u>MATH 244</u>.

   Qualitative theory of ordinary differential equations: Existence and Uniqueness Theorem, phase portraits, limit sets, stability of fixed points and periodic orbits, energy functions, Poincare-Bendixson Theorem, Poincare maps, bifurcations, attractors, chaos. 4 lectures.

2. <u>Required Background or Experience</u>

   Math 206 and Math 242, or Math 241 and Math 244.

3. <u>Learning Objectives</u>

   The student should:

   a. Identify fundamental differences between linear and nonlinear systems of differential equations.
   b. State and understand the Existence and Uniqueness Theorem for solutions.
   c. Identify fixed points and periodic orbits, and determine their stability.
   d. Construct and interpret phase portraits in one and two dimensions.
   e. Understand limit sets.
   f. Use energy functions to analyze systems.
   g. State and understand the Poincaré-Bendixson Theorem.
   h. Understand Poincaré maps.
   i. Identify elementary bifurcations.
   j. Understand and contrast characterizations of chaos.
   k. Use software to simulate and study continuous dynamical systems in one and two dimensions.
   l. Understand standard examples such as the harmonic oscillator, the pendulum, Lotka-Volterra equations, the van der Pol oscillator, and the Lorenz equations.

4. <u>Texts and References</u>

   To be chosen by the instructor. Suggested texts include:

   - Robinson, R. Clark, <u>An Introduction to Dynamical Systems: Continuous and Discrete</u>
   - Strogatz, Steven, <u>Nonlinear Dynamics and Chaos</u>
   - Hirsch, Smale & Devaney, <u>Differential Equations, Dynamical Systems & An Introduction to Chaos</u>

5. <u>Minimum Student Materials</u>

   Paper, pencils, and notebook.

6. <u>Minimum University Facilities</u>

Classroom with ample blackboard space for lectures, and a computer with a projector for demonstrations.

7. <u>Suggested Content and Method</u>

<u>Topic</u>

    a. **Geometric Approach to Differential Equations**

    b. **Linear Systems**
       1. Fundamental sets of solutions
       2. Constant coefficients: solutions and phase portraits
       3. Nonhomogeneous systems: time-dependent forcing

    c. **The Flow: Solutions of Nonlinear Equations**

    d. **Phase Portraits With Emphasis on Fixed Points**
       1. Stability of fixed points
       2. One-dimensional differential equations
       3. Two dimensions and nullclines
       4. Linearized stability of fixed points
       5. Competitive populations

    e. **Phase Portraits Using Energy and Other Test Functions**
       1. Predator-prey systems
       2. Undamped forces
       3. Lyapunov functions for damped systems
       4. Limit sets
       5. Gradient systems

    f. **Periodic Orbits**
       1. Definitions and examples
       2. Poincaré-Bendixson Theorem
       3. Self-excited oscillator
       4. Andronov-Hopf bifurcation
       5. Homoclinic bifurcation to a periodic orbit
       6. Change of area or volume by a flow
       7. Stability of periodic orbits and the Poincaré map

    g. **Chaotic Attractors**
       1. Attractors
       2. Chaos
       3. Lorenz system

8. <u>Methods of Assessment</u>

The primary methods of assessment are: essay examinations, quizzes and homework. Typically, there will be one or more hour-long examinations during the quarter, and a required comprehensive final examination. Students are required to show their work and

are graded not only on the correctness of their answers, but also on their understanding of the concepts and techniques.