

MATH 416 Differential Equations II

1. Catalog Description

MATH 416 Differential Equations II

4 units

Prerequisite: MATH 206 and MATH 242, or MATH 241 and MATH 244.

Qualitative theory of ordinary differential equations: Existence and Uniqueness Theorem, phase portraits, limit sets, stability of fixed points and periodic orbits, energy functions, Poincaré-Bendixson Theorem, Poincaré maps, bifurcations, attractors, chaos. 4 lectures.

2. Required Background or Experience

Math 206 and Math 242, or Math 241 and Math 244.

3. Learning Objectives

The student should:

- a. Identify fundamental differences between linear and nonlinear systems of differential equations.
- b. State and understand the Existence and Uniqueness Theorem for solutions.
- c. Identify fixed points and periodic orbits, and determine their stability.
- d. Construct and interpret phase portraits in one and two dimensions.
- e. Understand limit sets.
- f. Use energy functions to analyze systems.
- g. State and understand the Poincaré-Bendixson Theorem.
- h. Understand Poincaré maps.
- i. Identify elementary bifurcations.
- j. Understand and contrast characterizations of chaos.
- k. Use software to simulate and study continuous dynamical systems in one and two dimensions.
- l. Understand standard examples such as the harmonic oscillator, the pendulum, Lotka-Volterra equations, the van der Pol oscillator, and the Lorenz equations.

4. Texts and References

To be chosen by the instructor. Suggested texts include:

- Robinson, R. Clark, An Introduction to Dynamical Systems: Continuous and Discrete
- Strogatz, Steven, Nonlinear Dynamics and Chaos
- Hirsch, Smale & Devaney, Differential Equations, Dynamical Systems & An Introduction to Chaos

5. Minimum Student Materials

Paper, pencils, and notebook.

6. Minimum University Facilities

Classroom with ample blackboard space for lectures, and a computer with a projector for demonstrations.

7. Suggested Content and Method

Topic

a. **Geometric Approach to Differential Equations**

b. **Linear Systems**

1. Fundamental sets of solutions
2. Constant coefficients: solutions and phase portraits
3. Nonhomogeneous systems: time-dependent forcing

c. **The Flow: Solutions of Nonlinear Equations**

d. **Phase Portraits With Emphasis on Fixed Points**

1. Stability of fixed points
2. One-dimensional differential equations
3. Two dimensions and nullclines
4. Linearized stability of fixed points
5. Competitive populations

e. **Phase Portraits Using Energy and Other Test Functions**

1. Predator-prey systems
2. Undamped forces
3. Lyapunov functions for damped systems
4. Limit sets
5. Gradient systems

f. **Periodic Orbits**

1. Definitions and examples
2. Poincaré-Bendixson Theorem
3. Self-excited oscillator
4. Andronov-Hopf bifurcation
5. Homoclinic bifurcation to a periodic orbit
6. Change of area or volume by a flow
7. Stability of periodic orbits and the Poincaré map

g. **Chaotic Attractors**

1. Attractors
2. Chaos
3. Lorenz system

8. Methods of Assessment

Comprehensive final exam, mid-term exams or quizzes, homework.