MATH 400 Special Problems for Advanced Undergraduates

1. Catalog Description

MATH 400 Special Problems for Advanced Undergraduates **1-4 units**

Prerequisite: Junior standing and consent of department chair.

Individual investigation, research, studies, or surveys of selected problems. Total credit limited to 8 units.

2. Required Background or Experience

Students should have completed Math 242 or the equivalent and at least one upper division mathematics course.

3. Learning Objectives

Students should:

a. Demonstrate to the supervising instructor a level of mastery appropriate to their joint understanding as to the scope of the proposed undertakings.

4. Text and References

Any appropriate for the proposed undertakings.

5. Minimum Student Materials

None.

6. Minimum University Facilities

Appropriate library resources.

7. Content and Method

Before enrolling in Math 400, a student in consultation with their supervising instructor will complete, sign, and submit a proposal to the department chair for approval. The format for the proposal should follow the sample format provided. Once the proposal is approved, the student will in enroll in Math 400 for the number of units authorized.

Method

The supervising instructor shall schedule regular consultation periods with the student, at which time the student shall demonstrate satisfactory progress towards the goals outlined in the proposal they have jointly established.

8. Methods of Assessment

The method of evaluation will be consistent with the assessment methods contained in the approved proposal. See attached sample proposal.

MATH 400/500 Proposal

Probability with Martingales

`A copy of the LaTeX file is in the Math One Drive`

1 Reference

D. Williams. Probability with Martingales. *Cambridge University Press* (1991).

2 Course Outline

The goal of this course is the rigorous treatment of probability theory, with the aim of understanding and proving Doob's Martingale Convergence Theorem. If time permits we will also look at some application, for instance using Martingales to prove the (strong) law of large numbers – a theorem that describes the result of performing the same experiment a large number of times. Indeed, it states that the sample average converges almost surely to the expected value. For example, while a casino may lose money in a single spin of a roulette wheel, its earnings will tend towards a predictable percentage over a large number of spins. Other application could include option pricing (discrete Black-Scholes model) and the Mabinogion sheep problem.

3 Time-line

The following time-line for the course is proposed. (Chapter references refer to the book Probability with Martingales by D. Williams)

Weeks 1-2 Chapters 1-8. Here basic definitions and results from probability will be covered, such as random variables, events, independence, expectation and product spaces.

Weeks 3-4 Chapter 9. Definitions and properties of Conditional Expectation.

Weeks 5-7 Chapter 10. Definitions and properties of Martingales.

Week 8 Chapter 11: Doob's Martingale Convergence Theorem.

Week 9-10 Chapters 12-15: Applications of Martingales.

4 Assessment

It is proposed that each participant will give a weekly presentation (30-60 minutes) as well as the development of the following materials.

- A brief and well-organized outline of the main definitions and results of Chapters 1-8, including proofs of key results, for instance, the Borel-Cantelli Lemma, Kolmogorov's 0-1 law, the Weierstraß approximation theorem and Fubini's theorem. Here the participants will also provide a dictionary between terms in probability theory and measure theory.
- Solutions for a number of selected problems from the Chapters 1–8.
- All problems associated with Chapters 9-11.

5 Number of units

2 Units

Participants

Warren Ambrose .....

Paul Halmos .....

Supervisor

Prof. Joseph Doob .....

Chair of Mathematics

Prof. Joseph Walsh .....