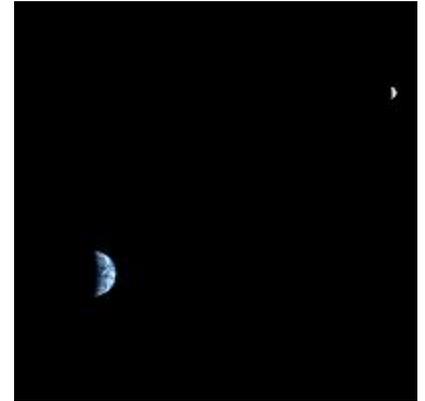


Tidal acceleration

Tidal acceleration is an effect of the tidal forces between an orbiting natural satellite (e.g. the Moon), and the primary planet that it orbits (e.g. Earth). The acceleration causes a gradual recession of a satellite in a prograde orbit away from the primary, and a corresponding slowdown of the primary's rotation. The process eventually leads to tidal locking, usually of the smaller first, and later the larger body. The Earth–Moon system is the best studied case.

The similar process of **tidal deceleration** occurs for satellites that have an orbital period that is shorter than the primary's rotational period, or that orbit in a retrograde direction.

The naming is somewhat confusing, because the speed of the satellite relative to the body it orbits is *decreased* as a result of tidal acceleration, and *increased* as a result of tidal deceleration.



A picture of Earth and the Moon from Mars. The presence of the moon (which has about 1/81 the mass of Earth), is slowing Earth's rotation and extending the day by about 2 milliseconds every 100 years.

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Earth–Moon system

Discovery history of the secular acceleration

Edmond Halley was the first to suggest, in 1695,^[1] that the mean motion of the Moon was apparently getting faster, by comparison with ancient eclipse observations, but he gave no data. (It was not yet known in Halley's time that what is actually occurring includes a slowing-down of Earth's rate of rotation: see also Ephemeris time – History. When measured as a function of mean solar time rather than uniform time, the effect appears as a positive acceleration.) In 1749 Richard Dunthorne confirmed Halley's suspicion after re-examining ancient records, and produced the first quantitative estimate for the size of this apparent effect:^[2] a centurial rate of +10" (arcseconds) in lunar longitude, which is a surprisingly accurate result for its time, not differing greatly from values assessed later, e.g. in 1786 by de Lalande,^[3] and to compare with values from about 10" to nearly 13" being derived about a century later^{[4][5]}

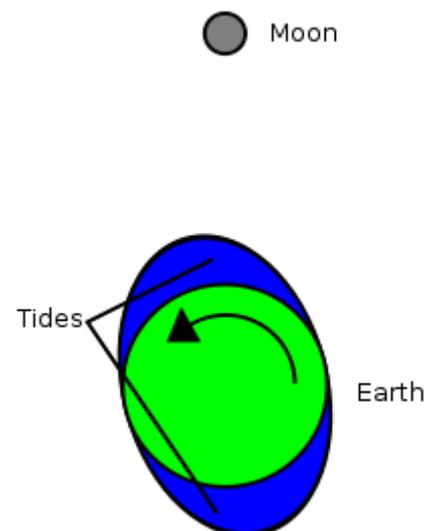
Pierre-Simon Laplace produced in 1786 a theoretical analysis giving a basis on which the Moon's mean motion should accelerate in response to perturbational changes in the eccentricity of the orbit of Earth around the Sun. Laplace's initial computation accounted for the whole effect, thus seeming to tie up the theory neatly with both modern and ancient observations.^[6]

However, in 1854, John Couch Adams caused the question to be re-opened by finding an error in Laplace's computations: it turned out that only about half of the Moon's apparent acceleration could be accounted for on Laplace's basis by the change in Earth's orbital eccentricity.^[7] Adams's finding provoked a sharp astronomical controversy that lasted some years, but the correctness of his result, agreed upon by other mathematical astronomers including C. E. Delaunay, was eventually accepted.^[8] The question depended on correct analysis of the lunar motions, and received a further complication with another discovery, around the same time, that another significant long-term perturbation that had been calculated for the Moon (supposedly due to the action of Venus) was also in error, was found on re-examination to be almost negligible, and practically had to disappear from the theory. A part of the answer was suggested independently in the 1860s by Delaunay and by William Ferrel: tidal retardation of Earth's rotation rate was lengthening the unit of time and causing a lunar acceleration that was only apparent.^[9]

It took some time for the astronomical community to accept the reality and the scale of tidal effects. But eventually it became clear that three effects are involved, when measured in terms of mean solar time. Beside the effects of perturbational changes in Earth's orbital eccentricity, as found by Laplace and corrected by Adams, there are two tidal effects (a combination first suggested by Emmanuel Liais). First there is a real retardation of the Moon's angular rate of orbital motion, due to tidal exchange of angular momentum between Earth and Moon. This increases the Moon's angular momentum around Earth (and moves the Moon to a higher orbit with a lower orbital speed). Secondly there is an apparent increase in the Moon's angular rate of orbital motion (when measured in terms of mean solar time). This arises from Earth's loss of angular momentum and the consequent increase in length of day^[10]

Effects of Moon's gravity

Because the Moon's mass is a considerable fraction of that of Earth (about 1:81), the two bodies can be regarded as a double planet system, rather than as a planet with a satellite. The plane of the Moon's sorbit around Earth lies close to the plane of Earth's orbit around the Sun (the ecliptic), rather than in the plane perpendicular to the axis of rotation of Earth (the equator) as is usually the case with planetary satellites. The mass of the Moon is sufficiently large, and it is sufficiently close, to raise tides in the matter of Earth. In particular, the water of the oceans bulges out towards and away from the Moon. The average tidal bulge is synchronized with the Moon's orbit, and Earth rotates under this tidal bulge in just over a day. However, Earth's rotation drags the position of the tidal bulge ahead of the position directly under the Moon. As a consequence, there exists a substantial amount of mass in the bulge that is offset from the line through the centers of Earth and the Moon. Because of this offset, a portion of the gravitational pull between Earth's tidal bulges and the Moon is not parallel to the Earth–Moon line, i.e. there exists a torque between Earth and the Moon. Since the bulge nearer the moon pulls more strongly on it than the bulge further away, this torque boosts the Moon in its orbit and slows the rotation of Earth.



A diagram of the Earth–Moon system showing how the tidal bulge is pushed ahead by Earth's rotation. This offset bulge exerts a net torque on the Moon, boosting it while slowing Earth's rotation.

As a result of this process, the mean solar day, which is nominally 86,400 seconds long, is actually getting longer when measured in SI seconds with stable atomic clocks. (The SI second, when adopted, was already a little shorter than the current value of the second of mean solar time.^[11]) The small difference accumulates over time, which leads to an increasing difference between our clock time (Universal Time) on the one hand, and Atomic Time and Ephemeris Time on the other hand: see ΔT. This led to the introduction of the leap second in 1972^[12] to compensate for differences in the bases for time standardization.

In addition to the effect of the ocean tides, there is also a tidal acceleration due to flexing of Earth's crust, but this accounts for only about 4% of the total effect when expressed in terms of heat dissipation.^[13]

If other effects were ignored, tidal acceleration would continue until the rotational period of Earth matched the orbital period of the Moon. At that time, the Moon would always be overhead of a single fixed place on Earth. Such a situation already exists in the Pluto–Charon system. However, the slowdown of Earth's rotation is not occurring fast enough for the rotation to lengthen to a month before other effects make this irrelevant: About 1 to 1.5 billion years from now, the continual increase of the Sun's radiation will likely cause Earth's oceans to vaporize,^[14] removing the bulk of the tidal friction and acceleration. Even without this, the slowdown to a month-long day would still not have been completed by 4.5 billion years from now when the Sun will probably evolve into a red giant and likely destroy both Earth and the Moon.^{[15][16]}

Tidal acceleration is one of the few examples in the dynamics of the Solar System of a so-called **secular perturbation** of an orbit, i.e. a perturbation that continuously increases with time and is not periodic. Up to a high order of approximation, mutual gravitational perturbations between major or minor planets only cause periodic variations in their orbits, that is, parameters oscillate between maximum and minimum values. The tidal effect gives rise to a quadratic term in the equations, which leads to unbounded growth. In the mathematical theories of the planetary orbits that form the basis of ephemerides, quadratic and higher order secular terms do occur, but these are mostly Taylor expansions of very long time periodic terms. The reason that tidal effects are different is that unlike distant gravitational perturbations, friction is an essential part of tidal acceleration, and leads to permanent loss of energy from the dynamic system in the form of heat. In other words, we do not have a Hamiltonian system here.

Angular momentum and energy

The gravitational torque between the Moon and the tidal bulge of Earth causes the Moon to be constantly promoted to a slightly higher orbit and Earth to be decelerated in its rotation. As in any physical process within an isolated system, total energy and angular momentum are conserved. Effectively, energy and angular momentum are transferred from the rotation of Earth to the orbital motion of the Moon (however, most of the energy lost by Earth (−3.321 TW) is converted to heat by frictional losses in the oceans and their interaction with the solid Earth, and only about 1/30th (+0.121 TW) is transferred to the Moon). The Moon moves farther away from Earth (+38.247±0.004 mm/y), so its potential energy (in Earth's gravity well) increases. It stays in orbit, and from Kepler's 3rd law it follows that its angular velocity actually decreases, so the tidal action on the Moon actually causes an angular deceleration, i.e. a negative acceleration (−25.858±0.003"/century²) of its rotation around Earth. The actual speed of the Moon also decreases. Although its kinetic energy decreases, its potential energy increases by a larger amount.

The rotational angular momentum of Earth decreases and consequently the length of the day increases. The *net* tide raised on Earth by the Moon is dragged ahead of the Moon by Earth's much faster rotation. **Tidal friction** is required to drag and maintain the bulge ahead of the Moon, and it dissipates the excess energy of the exchange of rotational and orbital energy between Earth and the Moon as heat. If the friction and heat dissipation were not present, the Moon's gravitational force on the tidal bulge would rapidly (within two days) bring the tide back into synchronization with the Moon, and the Moon would no longer recede. Most of the dissipation occurs in a turbulent bottom boundary layer in shallow seas such as the European Shelf around the British Isles, the Patagonian Shelf off Argentina, and the Bering Sea.^[17]

The dissipation of energy by tidal friction averages about 3.75 terawatts, of which 2.5 terawatts are from the principal M₂ lunar component and the remainder from other components, both lunar and solar^[18]

An *equilibrium tidal bulge* does not really exist on Earth because the continents do not allow this mathematical solution to take place. Oceanic tides actually rotate around the ocean basins as vast gyres around several amphidromic points where no tide exists. The Moon pulls on each individual undulation as Earth rotates—some undulations are ahead of the Moon, others are behind it, whereas

still others are on either side. The "bulges" that actually do exist for the Moon to pull on (and which pull on the Moon) are the net result of integrating the actual undulations over all the world's oceans. Earth's *net* (or *equivalent*) equilibrium tide has an amplitude of only 3.23 cm, which is totally swamped by oceanic tides that can exceed one metre.

Historical evidence

This mechanism has been working for 4.5 billion years, since oceans first formed on Earth. There is geological and paleontological evidence that Earth rotated faster and that the Moon was closer to Earth in the remote past. *Tidal rhythmites* are alternating layers of sand and silt laid down offshore from estuaries having great tidal flows. Daily, monthly and seasonal cycles can be found in the deposits. This geological record is consistent with these conditions 620 million years ago: the day was 21.9 ± 0.4 hours, and there were 13.1 ± 0.1 synodic months/year and 400 ± 7 solar days/year. The average recession rate of the Moon between then and now has been 2.17 ± 0.31 cm/year, which is about half the present rate. The present high rate may be due to near resonance between natural ocean frequencies and tidal frequencies.^[19]

Quantitative description of the Earth–Moon case

The motion of the Moon can be followed with an accuracy of a few centimeters by lunar laser ranging (LLR). Laser pulses are bounced off mirrors on the surface of the moon, emplaced during the Apollo missions of 1969 to 1972 and by Lunokhod 2 in 1973.^{[20][21]} Measuring the return time of the pulse yields a very accurate measure of the distance. These measurements are fitted to the equations of motion. This yields numerical values for the Moon's secular deceleration, i.e. negative acceleration, in longitude and the rate of change of the semimajor axis of the Earth–Moon ellipse. From the period 1970–2012, the results are:

$$\begin{aligned} & -25.82 \pm 0.03 \text{ arcsecond/century}^2 \text{ in ecliptic longitude}^{[22]} \\ & +38.08 \pm 0.04 \text{ mm/yr in the mean Earth–Moon distance}^{[22]} \end{aligned}$$

This is consistent with results from satellite laser ranging (SLR), a similar technique applied to artificial satellites orbiting Earth, which yields a model for the gravitational field of Earth, including that of the tides. The model accurately predicts the changes in the motion of the Moon.

Finally, ancient observations of solar eclipses give fairly accurate positions for the Moon at those moments. Studies of these observations give results consistent with the value quoted above.^[23]

The other consequence of tidal acceleration is the deceleration of the rotation of Earth. The rotation of Earth is somewhat erratic on all time scales (from hours to centuries) due to various causes.^[24] The small tidal effect cannot be observed in a short period, but the cumulative effect on Earth's rotation as measured with a stable clock (ephemeris time, atomic time) of a shortfall of even a few milliseconds every day becomes readily noticeable in a few centuries. Since some event in the remote past, more days and hours have passed (as measured in full rotations of Earth) (Universal Time) than would be measured by stable clocks calibrated to the present, longer length of the day (ephemeris time). This is known as ΔT . Recent values can be obtained from the International Earth Rotation and Reference Systems Service (IERS).^[25] A table of the actual length of the day in the past few centuries is also available.^[26]

From the observed change in the Moon's orbit, the corresponding change in the length of the day can be computed:

$$+2.3 \text{ ms/century}$$

However, from historical records over the past 2700 years the following average value is found:

$$+1.70 \pm 0.05 \text{ ms/century}^{[27][28]}$$

The corresponding cumulative value is a parabola having a coefficient of T^2 (time in centuries squared) of:

$$\Delta T = +31 \text{ s/century}^2$$

Opposing the tidal deceleration of Earth is a mechanism that is in fact accelerating the rotation. Earth is not a sphere, but rather an ellipsoid that is flattened at the poles. SLR has shown that this flattening is decreasing. The explanation is that during the ice age large masses of ice collected at the poles, and depressed the underlying rocks. The ice mass started disappearing over 10000 years ago, but Earth's crust is still not in hydrostatic equilibrium and is still rebounding (the relaxation time is estimated to be about 4000 years). As a consequence, the polar diameter of Earth increases, and the equatorial diameter decreases (Earth's volume must remain the same). This means that mass moves closer to the rotation axis of Earth, and that Earth's moment of inertia decreases. This process alone leads to an increase of the rotation rate (phenomenon of a spinning figure skater who spins ever faster as they retract their arms). From the observed change in the moment of inertia the acceleration of rotation can be computed: the average value over the historical period must have been about -0.6 ms/century. This largely explains the historical observations.

Other cases of tidal acceleration

Most natural satellites of the planets undergo tidal acceleration to some degree (usually small), except for the two classes of tidally decelerated bodies. In most cases, however, the effect is small enough that even after billions of years most satellites will not actually be lost. The effect is probably most pronounced for Mars's second moon Deimos, which may become an Earth-crossing asteroid after it leaks out of Mars's grip. The effect also arises between different components in a binary star.^[29]

Tidal deceleration

This comes in two varieties:

1. *Fast satellites*: Some inner moons of the giant planets and Phobos orbit within the synchronous orbit radius so that their orbital period is shorter than their planet's rotation. In other words, they orbit their planet faster than the planet rotates. In this case the tidal bulges raised by the moon on their planet lag behind the moon, and act to ~~de~~accelerate it in its orbit. The net effect is a decay of that moon's orbit as it gradually spirals towards the planet. The planet's rotation also speeds up slightly in the process. In the distant future these moons will strike the planet or cross within their Roche limit and be tidally disrupted into fragments. However, all such moons in the Solar System are very small bodies and the tidal bulges raised by them on the planet are also small, so the effect is usually weak and the orbit decays slowly. The moons affected are:

- *Around Mars*: Phobos
- *Around Jupiter*: Metis and Adrastea
- *Around Saturn*: none, except for the ring particles (like Jupiter, Saturn is a very rapid rotator but has no satellites close enough)
- *Around Uranus*: Cordelia, Ophelia, Bianca, Cressida, Desdemona, Juliet, Portia, Rosalind, Cupid, Belinda, and Perdita
- *Around Neptune*: Naiad, Thalassa, Despina, Galatea and Larissa

Some hypothesize that after the Sun becomes a red giant, its surface rotation will be much slower and it will cause tidal deceleration of any remaining planets.^[30]

2. *Retrograde satellites*: All retrograde satellites experience tidal deceleration to some degree because their orbital motion and their planet's rotation are in opposite directions, causing restoring forces from their tidal bulges. A difference to the previous "fast satellite" case here is that the planet's rotation is also slowed down rather than sped up (angular momentum is still conserved because in such a case the values for the planet's rotation and the moon's revolution have opposite signs). The only satellite in the Solar System for which this effect is non-negligible is Neptune's moon Triton. All the other retrograde satellites are on distant orbits and tidal forces between them and the planet are negligible.

Mercury and Venus are believed to have no satellites chiefly because any hypothetical satellite would have suffered deceleration long ago and crashed into the planets due to the very slow rotation speeds of both planets; in addition, Venus also has retrograde rotation.

Theory

Size of the tidal bulge

Neglecting axial tilt, the tidal force a satellite (such as the moon) exerts on a planet (such as earth) can be described by the variation of its gravitational force over the distance from it, when this force is considered as applied to a unit mass m :

$$\frac{\delta F}{\delta r} = -2 \frac{G m dm}{r^3}$$

where G is the universal gravitational constant, m is the satellite mass and r is the distance between the satellite and the planet.

Thus the satellite creates a disturbing potential on the planet, whose difference between the planet center and the closest (or farthest) point to the satellite is:

$$\Delta V = 2 \frac{G m dm A^2}{r^3}$$

where A is the planet radius.

The size of the tidal bulge created on the planet can be estimated as roughly the ratio between this disturbing potential and the planet surface gravity:

$$H \approx \frac{\Delta V}{G M dm / A^2} = 2 \frac{m A^4}{M r^3}$$

A more exact calculation gives:^[31]

$$H = \frac{15}{8} \frac{m A^4}{M r^3}$$

assuming we neglect a second order effect due to rigidity of the planet material.

For the moon-earth system ($m=7.3 \times 10^{22}$ kg, $M=6 \times 10^{24}$ kg, $A=6.4 \times 10^6$ m, $r=3.8 \times 10^8$), this gives 0.7 meters, close to the true value for ocean tides height (roughly one meter).

Note that two bulges are formed, one centered roughly around the point nearest to the satellite and the other centered roughly around the point farthest from it.

Torque

Due to planet rotation, the bulges lag somewhat behind the planet-satellite axis, which creates an angle α between the two. The size of this lag angle depends on inertia and (much more importantly) on dissipation forces (e.g. friction) exerted on the bulge.

The satellite applies different forces on the close and far bulges. The difference is roughly $\delta F / \delta r$ times the planet diameter, where we replace the unit mass in the calculation above with the approximate mass of each bulge, $\pi \rho A^2 H$ (where ρ is the mass density of the buldge):

$$\Delta F \approx \frac{\delta F}{\delta r} \cdot 2A \cdot \cos(\alpha) = 4\pi \frac{G m \rho A^3 H}{r^3} \cos(\alpha)$$

where we took into consideration the effect of the lag angle α .

In order to get a rough estimation for the torque exerted by the satellite on the planet, we need to multiply this difference with the lever length (which is the planet diameter), and with the sine of the lag angle, giving:

$$N \approx 8\pi \frac{G m \rho A^4 H}{r^3} \cos(\alpha) \sin(\alpha) = 4\pi \frac{G m \rho A^4 H}{r^3} \sin(2\alpha)$$

A more exact calculation adds a 2/5 factor due to the planet spherical form and gives:^[31]

$$N = \frac{8}{5}\pi \frac{G m \rho A^4 H}{r^3} \sin(2\alpha)$$

Inserting the value of H found above this is:

$$N = 3\pi \frac{G m^2 \rho A^8}{M r^6} \sin(2\alpha)$$

This can be written as:

$$N = \frac{9}{4} k \frac{G m^2 A^5}{r^6} \sin(2\alpha)$$

Where k is a factor related that can be expressed by Love numbers, taking into considerations non-uniformity in the planet mass density; corrections due to planet rigidity, neglected above, also enter here. For earth, most of the bulge is made of sea water and has no correction for rigidity, but its mass density is 0.18 the average earth mass density (1 g/cm³ vs. 5.5 g/cm³), so $k \approx 0.18$. The literature uses a close value of 0.2 ($\Leftarrow 2k_2/3$ ^[32])

A similar calculation can be done for the tides created on the planet by the sun. Here, m should be replaced by the mass sun, and r by the distance to the sun. Since α depends on the dissipation properties of earth, it is expected to be the same for both. The resulting torque is 20% that exerted by the moon.

Relation of the lag angle to energy dissipation

The work exerted by the satellite over the planet is created by a force F acting along the path of movement of a mass units moving in velocity u in the planet (in fact, in the bulge).

Forces and locations depend on the relative angle to the planet-satellite axis θ , that changes periodically with the angular momentum Ω . Since the force in the planet spherical coordinate system is symmetrical in the direction towards the satellite and in the opposite direction (it is outwards in both), the dependence is approximated as sinusoidal in 2θ . Thus the force exerted on a unit mass is of the form:

$$dF(t) = dF_0 \cos(2\theta) = dF_0 \cos(2\Omega t)$$

and the translation projected on the same direction is of the form:

$$\xi(t) = \xi_0 \cos(2(\theta - \alpha)) = \xi_0 \cos(2(\Omega t - \alpha))$$

due to the lag angle. The velocity component in the direction of the force is therefore:

$$u(t) = \frac{d\xi(t)}{dt} = -2\Omega \xi_0 \sin(2(\theta - \alpha))$$

And so the total work exerted over a unit mass during one cycle is^[32]

$$\int_0^{\pi/\Omega} \vec{dF}(t) \vec{u}(t) dt = -2\Omega dF_0 \xi_0 \int_0^{\pi/\Omega} \cos(2\Omega t) \sin(2(\Omega t - \alpha)) dt = -\pi dF_0 \xi_0 \sin(2\alpha)$$

In fact, almost all of this is dissipated (e.g. as friction), as explained below

Looking now at the total energy from the satellite potential in one of the bulges, this is equal to the total work performed on this in a quarter of the total angular range, i.e. from zero to maximal displacement:

$$\begin{aligned}
E^* &= \int_{-\pi/4\Omega+\alpha/\Omega}^{\alpha/\Omega} \vec{dF}(t) \vec{u}(t) dt = -2\Omega dF_0 \xi_0 \int_{-\pi/4\Omega+\alpha/\Omega}^{\alpha/\Omega} \cos(2\Omega t) \sin(2(\Omega t - \alpha)) dt \\
&= -dF_0 \xi_0 \int_{-\pi/2}^0 \cos(z + 2\alpha) \sin(z) dz \approx \frac{1}{2} dF_0 \xi_0
\end{aligned}$$

where we have defined $z = 2(\Omega t - \alpha)$, and approximated for small α in the last equality thus neglecting it.

The fraction of energy dissipated in each cycle is represented by the effective specific dissipation function, denoted by Q^{-1} and defined as the total dissipation in one cycle divided by $2\pi E^*$. This gives:^[32]

$$Q^{-1} = \sin(2\alpha)$$

The value of this is estimated as 1/13 for earth, where the bulge is mainly liquid, 10^{-1} - 10^{-2} for the other inner planets and the moon, where the bulge is mainly solid, and as 10^3 - 10^{-5} for the outer, mostly gaseous planets.^{[31][32]}

With this value for earth at hand, the torque can be calculated to be 4.4×10^{16} N m, only 13% above the measured value of 3.9×10^{16} N m.^[32]

Note that in the distant past, the value of $k \cdot \sin(2\alpha)$ for the earth - moon system was probably somewhat smaller^[32]

Retardation of the planet's rotation

Again neglecting axial tilt, The change over time in the planet angular momentum L is equal to the torque. L in turn is the product of the angular velocity Ω with the moment of inertia I .

For a spherical planet of approximately uniform mass density, $I = f M A^2$, where f is a factor depending on the planet structure; a spherical planet of uniform density has $f = 2/5 = 0.4$. Since the angular momentum This gives:

$$\frac{d\Omega}{dt} = \frac{dL}{I dt} = \frac{N}{I} = \frac{45}{8} k \frac{G m^2 A^3}{M r^6} \sin(2\alpha)$$

Since the earth density is larger at depth, its moment of inertia is somewhat smaller with $f = 0.33$.^[33]

For the earth-moon system, taking $\sin(2\alpha)$ of 1/13 and $k=0.2$, we get $4.5 \times 10^{-22} \text{sec}^{-2}$. For a 24-hour day, this is equivalent to 17 seconds in 1 million years, or 1 hour (i.e. lengthening of the day by 1 hour) in 210 million years. Due to the additional 20% effect of the sun, the day lengthens by 1 hour in approximately 180 million years.

A similar calculation shows that the earth had exerted angular momentum through tidal friction on the moon self-rotation, before this became tidally locked. At that period, one calculates the change in the moon angular momentum ω in the same manner as for Ω above, except that m and M should be switched, and A should be replaced by the moon radius $a = 1.7 \times 10^6$ meter. Taking $\sin(2\alpha)$ of 10^{-1} - 10^{-2} as for the solid planets and $k=1$, this gives $d\omega/dt = 3 \times 10^{-17}$ - $3 \times 10^{-18} \text{sec}^{-2}$. For a 29.5-day long rotation period, this is equivalent to 1.5-15 minutes in 1 year, or 1 day in 10^2 - 10^3 years. Thus in astronomical timescales, the moon became tidally locked very fast.

Effect on the satellite motion around the planet

Due to conservation of angular momentum, a torque of the same size as the one exerted by the satellite and of opposite direction is exerted by the planet on the satellite motion around the planet. Another effect, which will not be dealt with here, is the changes in the eccentricity and inclination of the orbit.

The moment of inertia of this motion is $m r^2$. However now r itself depends on the angular velocity which we denote here n : according to Newtonian analysis of orbital motion

$$r^3 n^2 = GM$$

Thus the satellite orbit angular momentum l , satisfies (neglecting eccentricity):

$$l = m r^2 n = m \sqrt{GM} r^{1/2}$$

$$N = \frac{dL}{dt} = \frac{1}{2} m \sqrt{GM} r^{-1/2} \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{2r^{1/2}}{m \sqrt{GM}} N = \frac{9}{2} k \sqrt{\frac{G}{M}} \frac{m A^5}{r^{11/2}} \sin(2\alpha)$$

Additionally, since $n = \sqrt{GM} r^{-3/2}$, we have:

$$\frac{dn}{dt} = -\frac{3}{2} \sqrt{GM} r^{-5/2} \frac{dr}{dt} = \frac{27}{4} k G \frac{m A^5}{r^8} \sin(2\alpha)$$

Note that assuming all rotations are on the same direction and $\Omega > \omega$; as time passes, the angular momentum of the planet decreases and hence that of the satellite orbit increases. Due to its relation with the planet-satellite distance, the latter increases, so the angular velocity of the satellite orbit decreases.

For the earth-moon system, dr/dt gives 1.2×10^{-9} meter per second, or 3.7 cm per year. This is a 1% increase in the earth-moon distance in 100 million years. dn/dt is $1.3 \times 10^{-23} \text{sec}^{-2}$, and for a period of 29.5 days is equivalent to 7 minutes in 1 million years, or 1 day (i.e. lengthening of the lunar period in 1 day) in 210 million years.

Effect of the Sun

The sun-planet system has two tidal friction effects. One effect is that the sun creates a tidal friction in the planet, which decreases its spinning angular momentum and hence also increases its orbital angular momentum around the sun, hence increasing its distance and reducing its angular velocity (assuming the orbital angular velocity of the sun is smaller than that of the planet spinning; otherwise directions of change are opposite).

If M_S is the sun mass and D is the distance to it, then the rate of change of D is given, similar to the above calculation, by:

$$\frac{dD}{dt} = \frac{2D^{1/2}}{M \sqrt{GM_S}} N_S = \frac{9}{2} k \frac{\sqrt{G} M_S^{3/2} A^5}{M D^{11/2}} \sin(2\alpha)$$

The planet orbital angular velocity Ω_S , then changes as:

$$\frac{d\Omega_S}{dt} = -\frac{3}{2} \sqrt{GM_S} D^{-5/2} \frac{dD}{dt} = \frac{27}{4} k \frac{G M_S^2 A^5}{M D^8} \sin(2\alpha)$$

For the earth-sun system, this gives 1×10^{-13} meters per second, or 3 meters in 1 million years. This is a 1% increase in the earth-sun distance in half a billion years. The decrease in the earth orbital angular momentum is $2 \times 10^{-31} \text{sec}^2$, or equivalently for a 1-year period, 1 second in 1 billion years.

Another, relatively negligible, effect is that the planet creates tidal friction in the sun. This creates a change in the distance to the sun and the orbital angular velocity around it, as it does for the satellite in the satellite-planet system. Using the same equations but now for the planet-sun system, with A_S standing for the sun radius (7×10^8 meter), we have:

$$\frac{dD}{dt} = \frac{9}{2} k_S \sqrt{\frac{G}{M_S}} \frac{M A_S^5}{D^{11/2}} \sin(2\alpha_S)$$

$$\frac{d\Omega_S}{dt} = \frac{27}{4} k_S G \frac{M A_S^5}{D^8} \sin(2\alpha_S)$$

where k_S is a factor, presumably very small, due to the non-uniformity of mass densities of the sun. Assuming this factor times $\sin(2\alpha_S)$ to be not larger than what is found in the outer planets, i.e. 10^{-3} - 10^{-5} ,^[31] we have a negligible contribution from this effect.

A detailed calculation for the earth - moon system

Potential perturbation created by the moon on earth

The potential per mass unit that the moon creates on earth, whose center is located at distance r_0 from the moon along the z-axis, in the earth - moon rotating frame of reference and in coordinates centered at the earth center is:

$$\mathcal{W} = -\frac{Gm}{|\vec{r} - r_0 \hat{z}|} + \frac{1}{2} \omega^2 |\vec{r} - r_1 \hat{z}|^2$$

where r_1 is the distance from the moon to the center of mass of the earth - moon system, ω is the angular velocity of the earth around this point (the same as the lunar orbital angular velocity). The second term is the effective potential due to the centrifugal force of the earth.

We expand the potential in Taylor series around the point. The linear term must vanish (at least on average in time) since otherwise the force on the earth center would be non vanishing. Thus:

$$\begin{aligned} \mathcal{W} &= \frac{1}{2} \omega^2 (x^2 + y^2 + (z - r_1)^2) - \frac{Gm}{\sqrt{x^2 + y^2 + (z - r_0)^2}} \\ &= \text{const} + \frac{1}{2} \omega^2 (x^2 + y^2 + z^2) - \frac{Gm}{r_0^3} \left(z^2 - \frac{1}{2} (x^2 + y^2) \right) + \frac{Gm}{r_0^5} (\dots) + \dots \end{aligned}$$

Moving to spherical coordinates this gives:

$$\begin{aligned} \mathcal{W} &= \text{const} + \frac{1}{2} \omega^2 r^2 - \frac{Gm r^2}{r_0^3} \frac{1}{2} (3 \cos^2(\theta) - 1) + \frac{Gm}{r_0^5} (\dots) + \dots \\ &= \text{const} + \frac{1}{2} \omega^2 r^2 - Gm \sum_{n=2}^{\infty} \frac{r^n}{r_0^{n+1}} P_n(\cos \theta) \end{aligned}$$

where $P_n(\cos \theta)$ are the Legendre polynomials

The constant term has no mechanical importance, while the r^2 causes a fixed dilation, and is not directly involved in creating a torque.

Thus we focus on the other terms, whose sum we denote \mathcal{W}_{2+} , and mainly on the $P_2(\cos \theta)$ term which is the largest, as $\frac{r}{r_0}$ is at most the ratio of the earth radius to its distance from the moon, which is less than 2%.

Form of the bulge I: response to a perturbative potential

We treat the potential created by the moon as a perturbation to the earth's gravitational potential. Thus the height on earth at angles θ , φ is:

$$r(\theta, \varphi) = A + \delta(\theta, \varphi)$$

where $\delta \ll A$, and the amplitude of δ is proportional to the perturbation. We expand δ in Legendre polynomials, where the constant term (which stands for dilation) will be ignored as we are not interested in it. Thus:

$$\delta(\theta, \varphi) = \sum_{n=1}^{\infty} \delta_n P_n(\cos \theta)$$

where δ_n are unknown constants we would like to find.

We assume for the moment total equilibrium, as well as no rigidity on earth (e.g. as in a liquid earth). Therefore, its surface is equipotential, and so $V_E(r(\theta, \varphi)) + W_{2+}(r(\theta, \varphi))$ is constant, where $V_E(r)$ is the earth potential per unit mass. Since δ is proportional to W_{2+} , which is much smaller than V_E , This can be expanded in δ . Dropping non-linear terms we have:

$$\begin{aligned} const &= V_E(r(\theta, \varphi)) + W_{2+}(r(\theta, \varphi)) \approx V_E(A) + V_E'(A)\delta(\theta, \varphi) + W_{2+}(A) \\ const &= V_E'(A) \sum_{n=1}^{\infty} \delta_n P_n(\cos \theta) - Gm \sum_{n=2}^{\infty} \frac{A^n}{r_0^{n+1}} P_n(\cos \theta) \end{aligned}$$

Note that $V_E'(r) \equiv dV_E(r)/dr$ is the force per unit mass from earth's gravity. i.e. $V_E'(A)$ is just the gravitational acceleration g .

Since the Legendre polynomials are orthogonal, we may equate their coefficients on both sides of the equation, giving:

$$\begin{aligned} \delta_1 &= 0 \\ \delta_n &= \frac{GmA^n/r_0^{n+1}}{V_E'(A)} = \frac{GmA^n/r_0^{n+1}}{GM/A^2} = \frac{mA^{n+2}}{Mr_0^{n+1}}, \quad n \geq 2 \end{aligned}$$

Thus the height is the ratio between the perturbation potential and the force from the perturbed potential.

Form of the bulge II: the deformation creating a perturbative potential

So far we have neglected the fact that the deformation itself creates a perturbative potential. In order to account for this, we may calculate this perturbative potential, re-calculate the deformation and continue so iteratively

Let us assume the mass density is uniform. Since δ is much smaller than A , the deformation can be treated as a thin shell added to the mass of the earth, where the shell has a surface mass density $\rho \delta$ (and can also be negative), with ρ being the mass density (if mass density is not uniform, then the change of shape of the planet creates differences in mass distribution in all depth, and this has to be taken into account as well). Since the gravitation potential has the same form as the electric potential, this is a simple problem in electrostatics. For the analogous electrostatic problem, the potential created by the shell has the form:

$$\begin{aligned} \sum_{n=0}^{\infty} a_n r^n P_n(\cos \theta), \quad r \leq A \\ \sum_{n=0}^{\infty} a_n \frac{A^{2n+1}}{r^{n+1}} P_n(\cos \theta), \quad r \geq A \end{aligned}$$

Where the surface charge density is proportional to the discontinuity in the gradient of the potential:

$$\sigma(\theta) = \epsilon_0 \sum_{n=0}^{\infty} (2n + 1) a_n A^{n-1} P_n(\cos \theta)$$

ϵ_0 is the vacuum permittivity, a constant relevant to electrostatics, related to the equation $U(r) = \frac{1}{4\pi\epsilon_0} \frac{q^2}{r}$. The analogous equation in gravity is $U(r) = G \frac{m^2}{r}$, so if charge density is replaced with mass density, ϵ_0 should be replaced with $\frac{1}{4\pi G}$.

Thus in the gravitational problem we have:

$$\rho \sum_n \delta_n P_n(\cos \theta) = \frac{1}{4\pi G} \sum_n (2n+1) a_n A^{n-1} P_n(\cos \theta)$$

So that, again due to the orthogonality of Legendre polynomials:

$$a_n = 4\pi \frac{G\rho}{(2n+1)A^{n-1}} \delta_n$$

Thus the perturbative potential per mass unit for $r \geq A$ is:

$$\begin{aligned} & 4\pi G\rho A^{n+2} \sum_{n=0}^{\infty} \frac{1}{(2n+1)r^{n+1}} \delta_n P_n(\cos \theta) \\ &= 3 \frac{GM}{A^2} \sum_n \frac{A^{n+1}}{(2n+1)r^{n+1}} \delta_n P_n(\cos \theta) \end{aligned}$$

Note that since earth's mass density is in fact not uniform, this result must be multiplied by a factor that is roughly the ratio of the bulge mass density and the average earth mass, approximately 0.18. The actual factor is somewhat larger, since there is some deformation in the deeper solid layers of earth as well. Let us denote this factor by x . Rigidity also lowers x , though this is less relevant for most of the bulge, made of sea water

The deformation was created by the a perturbative potential of size $\mathcal{W}_{2+}(A) = \frac{GM}{A^2} \delta_n P_n(\cos \theta)$. Thus for each coefficient of $P_n(\cos \theta)$, the ratio of the original perturbative potential to that secondarily created by the deformation is:

$$c_n \equiv \frac{3x}{2n+1}$$

with $x=1$ for perfectly a non-rigid uniform planet.

This secondary perturbative potential creates another deformation which again creates a perturbative potential and so on ad infinitum, so that the total deformation is of the size:

$$\sum_n \sum_{k=0}^{\infty} c_n^k h_n P_n(\cos \theta) = \sum_n \frac{1}{1-c_n} \delta_n P_n(\cos \theta)$$

For each mode, the ratio to δ_n , the naive estimation of the deformation, is $\frac{1}{1-c_n}$ and is denoted as Love number h_n . For a perfectly a non-rigid uniform planet (e.g. a liquid earth of non-compressible liquid), this is equal to $\frac{2n+1}{2}$, and for the main mode of $n=2$, it is 5/2.

Similarly, n -th mode of the tidal perturbative potential per unit mass created by earth at $r = A$ is the Love number k_n times the corresponding term in the original lunar tidal perturbative potential, where for a uniform mass density zero rigidity planet k_n is:

$$k_n = \sum_{k=1}^{\infty} c_n^k = \frac{c_n}{1-c_n}$$

For a perfectly a non-rigid uniform planet (e.g. a liquid earth of non-compressible liquid), this is equal to 3/2. In fact, for the main mode of $n=2$, the real value for earth is a fifth of it, namely $k_2 = 0.3$ ^[32] (which fits $c_2 = 0.23$ or $x = 0.38$, roughly twice the density ratios of 0.18).

Calculation of the torque

Instead of calculating the torque exerted by the moon on the earth deformation, we calculate the reciprocal torque exerted by the earth deformation on the moon; both must be equal.

The potential created by the earth bulge is the perturbative potential we have discussed above. Per unit mass, for $r = A$, this is the same as the lunar perturbative potential creating the bulge, with each mode multiplied by k_n , with the $n=2$ mode far dominating the potential. Thus at $r=A$ the bulge perturbative potential per unit mass is^[32]

$$U(r = A, \theta) = -Gm \sum_{n=2}^{\infty} k_n \frac{A^n}{r_0^{n+1}} P_n(\cos \theta)$$

since the n -th mode it drops off as $r^{-(n+1)}$ for $r > A$, we have outside earth:

$$U(r, \theta) = -Gm \sum_{n=2}^{\infty} k_n \frac{A^n}{r_0^{n+1}} \frac{A^n + 1}{r^{n+1}} P_n(\cos \theta)$$

However, the bulge actually lags at an angle α with respect to the direction to the moon due to earth's rotation. Thus we have:

$$U = -Gm \sum_{n=2}^{\infty} k_n \frac{A^{2n+1}}{r_0^{n+1} r^{n+1}} P_n(\cos(\theta - \alpha))$$

The moon is at $r=r_0$, $\theta=0$. Thus the potential per unit mass at the moon is:

$$\begin{aligned} U(r = r_0, \theta = 0) &= -Gm \sum_{n=2}^{\infty} k_n \frac{A^{2n+1}}{r_0^{2n+2}} P_n(\cos(\alpha)) \approx -Gmk_2 \frac{A^5}{r_0^6} P_2(\cos \alpha) \\ &= -Gmk_2 \frac{A^5}{r_0^6} \cdot \frac{1}{2} (3 \cos^2 \alpha - 1) \end{aligned}$$

Neglecting eccentricity and axial tilt, We get the torque exerted by the bulge on the moon by multiplying U with the moon's mass m , and differentiating with respect to θ at the moon location. This is equivalent to differentiating $mU(r = r_0, \theta = 0)$ with respect to α ,^[32] and gives:

$$N = m \frac{dU(r = r_0, \theta = 0)}{d\alpha} = \frac{3}{2} Gm^2 k_2 \frac{A^5}{r_0^6} \cdot \sin(2\alpha)$$

This is the same formula used above, with $r=r_0$ and k there defined as $2k_2/3$.

See also

- [Tidal locking](#)
- [Tidal force](#)
- [Tides](#)
- [Tidal heating](#)

References

1. E Halley (1695), "Some Account of the Ancient State of the City of Palmyra, with Short Remarks upon the Inscriptions Found there" (https://www.jstor.org/stable/102291?seq=1#page_scan_tab_contents) *Phil. Trans.*, vol.19 (1695–1697), pages 160–175; esp. at pages 174–175. (see also transcription using a modern font [here \(https://book.google.co.uk/books?id=b-Q_AAAAYAAJ&pg=PA65&lpg=PA65\)](https://book.google.co.uk/books?id=b-Q_AAAAYAAJ&pg=PA65&lpg=PA65))

2. Richard Dunthorne (1749), "A Letter from the Rev Mr. Richard Dunthorne to the Reverend Mr Richard Mason F. R. S. and Keeper of the Wood-Wardian Museum at Cambridge, concerning the Acceleration of the Moon" (<http://rstl.royalsocietypublishing.org/content/46/492/162.full.pdf>) *Philosophical Transactions*, Vol. 46 (1749–1750) #492, pp.162–172; also given in *Philosophical Transactions (abridgements) (1809)*, vol.9 (for 1744–49), p669–675(<https://archive.org/stream/philosophicaltra09royarich#page/669/mode/2up>) as "On the Acceleration of the Moon, by the Rev Richard Dunthorne".
3. J de Lalande (1786): "Sur les equations seculaires du soleil et de la lune" (<http://gallica.bnf.fr/ark:/12148/bpt6k3585j/f484.double>), *Memoires de l'Academie Royale des Sciences*, pp.390–397, at page 395.
4. J D North (2008), "Cosmos: an illustrated history of astronomy and cosmology", (University of Chicago Press, 2008) chapter 14, at page 454 (<https://books.google.com/books?id=qq8Luhs7rTUC&pg=PA454>).
5. See also P Puiseux (1879), "Sur l'acceleration seculaire du mouvement de la Lune" (http://archive.numdam.org/article/ASENS_1879_2_8_361_0.pdf) *Annales Scientifiques de l'Ecole Normale Superieure*, 2nd series vol.8 (1879), pp.361–444, at pages 361–365.
6. Britton, John (1992). *Models and Precision: The Quality of Ptolemy's Observations and Parameters* Garland Publishing Inc. p. 157. ISBN 978-0815302155
7. Adams, J C (1853). "On the Secular Variation of the Moon's Mean Motion" (<http://rstl.royalsocietypublishing.org/content/143/397.full.pdf>) (PDF). *Phil. Trans. R. Soc. Lond* **143**: 397–406. doi:10.1098/rstl.1853.0017 (<https://doi.org/10.1098/rstl.1853.0017>)
8. D. E. Cartwright (2001), "Tides: a scientific history" (<https://books.google.com/books?id=78bE5U7TmIC&pg=PA144>), (Cambridge University Press 2001), chapter 10, section: "Lunar acceleration, earth retardation and tidal friction" at pages 144–146.
9. Khalid, M.; Sultana, M.; Zaidi, F (2014). "Delta: Polynomial Approximation of Time Period 1620–2013" (<https://www.indawi.com/journals/jas/2014/480964/abs>) *Journal of Astrophysics* **2014**. doi:10.1155/2014/480964 (<https://doi.org/10.1155/2014/480964>) Retrieved 3 June 2017.
10. F R Stephenson (2002), "Harold Jeffreys Lecture 2002: Historical eclipses and Earth's rotation" (<http://articles.adsabs.harvard.edu/full/2003A%26G....44b..22S>) in *Astronomy & Geophysics* vol.44 (2002), pp. 2.22–2.27.
11. : (1) In McCarthy, D D; Hackman, C; Nelson, R A (2008). "The Physical Basis of the Leap Second" (<http://www.dtic.mil/get-tr-doc/pdf?AD=ADA489427>) *Astronomical Journal* **136**: 1906–1908. Bibcode:2008AJ....136.1906M (<http://adsabs.harvard.edu/abs/2008AJ....136.1906M>) doi:10.1088/0004-6256/136/5/1906 (<https://doi.org/10.1088/0004-6256/136/5/1906>) it is stated (page 1908), that "the SI second is equivalent to an older measure of the second of UT1, which was too small to start with and further as the duration of the UT1 second increases, the discrepancy widens." : (2) In the late 1950s, the cesium standard was used to measure both the current mean length of the second of mean solar time (UT2) (result: 9192631830 cycles) and also the second of ephemeris time (ET) (result: 9192631770±20 cycles), see "Time Scales", by L. Essen (<http://www.leapsecond.com/history/1968-Metrologia-v4-n4-Essen.pdf>), in *Metrologia*, vol.4 (1968), pp.161–165, on p.162. As is well known, the 9192631770 figure was chosen for the SI second. L Essen in the same 1968 article (p.162) stated that this "seemed reasonable in view of the variations in UT2".
12. "What's a Leap Second" (<http://www.timeanddate.com/time/leapseconds.html>). *Timeanddate.com*
13. Munk (1997). "Once again: once again—tidal friction" (<http://www.sciencedirect.com/science/article/pii/S0079661197000219>). *Progress in Oceanography* **40**: 7–35. Bibcode:1997PrOce..40....7M (<http://adsabs.harvard.edu/abs/1997PrOce..40....7M>) doi:10.1016/S0079-6611(97)00021-9 ([https://doi.org/10.1016/S0079-6611\(97\)00021-9](https://doi.org/10.1016/S0079-6611(97)00021-9))
14. Puneet Kollipara (22 January 2014), "Earth Won't Die as Soon as Thought" (<http://www.sciencemag.org/news/2014/01/earth-wont-die-soon-thought>) *Science*.
15. Murray, C.D.; Dermott, Stanley F (1999). *Solar System Dynamics* Cambridge University Press. p. 184. ISBN 978-0-521-57295-8
16. Dickinson, Terence (1993). *From the Big Bang to Planet X* Camden East, Ontario: Camden House pp. 79–81. ISBN 978-0-921820-71-0
17. Munk, Walter (1997). "Once again: once again—tidal friction". *Progress in Oceanography* **40** (1–4): 7–35. Bibcode:1997PrOce..40....7M (<http://adsabs.harvard.edu/abs/1997PrOce..40....7M>) doi:10.1016/S0079-6611(97)00021-9 ([https://doi.org/10.1016/S0079-6611\(97\)00021-9](https://doi.org/10.1016/S0079-6611(97)00021-9))
18. Munk, W.; Wunsch, C (1998). "Abyssal recipes II: energetics of tidal and wind mixing" *Deep-Sea Research Part I* **45** (12): 1977–2010. Bibcode:1998DSRI...45.1977M (<http://adsabs.harvard.edu/abs/1998DSRI...45.1977M>) doi:10.1016/S0967-0637(98)00070-3 ([https://doi.org/10.1016/S0967-0637\(98\)00070-3](https://doi.org/10.1016/S0967-0637(98)00070-3))

19. Williams, George E. (2000). "Geological constraints on the Precambrian history of Earth's rotation and the Moon's orbit". *Reviews of Geophysics* **38** (1): 37–60. Bibcode:2000RvGeo..38...37W(<http://adsabs.harvard.edu/abs/2000RvGeo..38...37W>) doi:10.1029/1999RG900016(<https://doi.org/10.1029%2F1999RG900016>)
20. Most laser pulses, 78%, are to the Apollo 15 site. See Williams, et al. (2008), p. 5.
21. Another reflector emplaced by Lunokhod 1 in 1970 is no longer functioning. See [Lunar Lost & Found: The Search for Old Spacecraft by Leonard David](http://www.space.com/scienceastronomy/060327_mystery_mondayhtml)(http://www.space.com/scienceastronomy/060327_mystery_mondayhtml)
22. J.G. Williams, D.H. Boggs and W.M. Folkner (2013). DE430 Lunar Orbit, Physical Librations, and Surface Coordinates (http://proba2.sidc.be/aux/data/spice/docs/DE430_Lunar_Ephemeris_and_Orientation.pdf)10. "These derived values depend on a theory which is not accurate to the number of digits given." See also : Chapront, Chapront-Touzé, Francou (2002). [A new determination of lunar orbital parameters, precession constant and tidal acceleration from LLR measurements](http://www.aanda.org/articles/aa/pdf/2002/0/aa2201.pdf)(<http://www.aanda.org/articles/aa/pdf/2002/0/aa2201.pdf>)
23. F.R. Stephenson, L.V. Morrison (1995): 'Long-term fluctuations in the Earth's rotation: 700 BC to AD 1990'(<http://rst.a.royalsocietypublishing.org/content/351/1695/165.full.pdf>) *Philosophical Transactions of the Royal Society of London Series A*, pp.165–202. doi:10.1098/rsta.1995.0028(<https://doi.org/10.1098%2Frsta.1995.0028>)
24. Jean O. Dickey (1995): "Earth Rotation Variations from Hours to Centuries". In: I. Appenzeller (ed.):*Highlights of Astronomy*. Vol. 10 pp.17..44.
25. http://www.iers.org/nn_10910/ERS/EN/Science/EarthRotation/UT1-TAI.html
26. LOD (<http://www.iers.org/iers/earth/rotation/ut1lodtable3.html>) Archived (<https://web.archive.org/web/20010908035636/http://www.iers.org/iers/earth/rotation/ut1lodtable3.html>) 2001-09-08 at the [Wayback Machine](http://www.iers.org/iers/earth/rotation/ut1lodtable3.html)
27. Dickey, Jean O.; Bender, PL; Faller, JE; Newhall, XX; Rinklefs, RL; Ries, JG; Shelus, PJ; Willet, C; et al. (1994). "Lunar Laser ranging: a continuing legacy of the Apollo program"(<http://www.physics.ucsd.edu/~tmurphy/apdb/doc/Dickey.pdf>) (PDF). *Science*. **265** (5171): 482–90. Bibcode:1994Sci...265..482D(<http://adsabs.harvard.edu/abs/1994Sci...265..482D>) doi:10.1126/science.265.5171.482(<https://doi.org/10.1126%2Fscience.265.5171.482>) PMID 17781305 (<https://www.ncbi.nlm.nih.gov/pubmed/17781305>).
28. F.R. Stephenson (1997):*Historical Eclipses and Earth's Rotation*(<https://books.google.com/books?id=DTb4DDuJNa4C>). Cambridge Univ Press.
29. Zahn, J.-P. (1977). "Tidal Friction in Close Binary Stars". *Astron. Astrophys* **57**: 383–394. Bibcode:1977A&A....57..383Z(<http://adsabs.harvard.edu/abs/1977A&A....57..383Z>)
30. Schröder, K.-P.; Smith, R.C. (2008). "Distant future of the Sun and Earth revisited" *Monthly Notices of the Royal Astronomical Society* **386** (1): 155–163. arXiv:0801.4031 (<https://arxiv.org/abs/0801.4031>) Bibcode:2008MNRAS.386..155S(<http://adsabs.harvard.edu/abs/2008MNRAS.386..155S>)doi:10.1111/j.1365-2966.2008.13022.x(<https://doi.org/10.1111%2Fj.1365-2966.2008.13022.x>)See also Palmer, J. (2008). "Hope dims that Earth will survive Sun's death"(<https://www.newscientist.com/article/dn1330-hope-dims-that-earth-will-survive-suns-death.html>). *New Scientist*. Retrieved 2008-03-24.
31. Goldreich, P., & Soter, S. (1966). Q in the Sdar System. *Icarus*, 5(1-6), 375-389.(https://websites.pmc.ucsc.edu/~pkoch/EART_206/09-0127/Goldreich%20&%20Soter%2066%20Icarus%205-375.pdf)
32. MacDonald, G. J. (1964). Tidal friction. *Reviews of Geophysics*, 2(3), 467-541.
33. Introduction to Earth Sciences I, Columbia university(http://www.columbia.edu/itc/ldeo/earthsci/topic2/topic2_1.html)

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