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Newton's law of universal gravitation

Newton's law of universal gravitation states that every particle attracts every other particle in the universe with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers.[note 1] This is a general physical law derived from empirical observations by what Isaac Newton called inductive reasoning.[1] It is a part of classical mechanics and was formulated in Newton's work *Philosophiæ Naturalis Principia Mathematica* ("the *Principia*"), first published on 5 July 1687. When Newton presented Book 1 of the unpublished text in April 1686 to the Royal Society, Robert Hooke made a claim that Newton had obtained the inverse square law from him.

In today's language, the law states that every point mass attracts every other point mass by a force acting along the line intersecting the two points. The force is proportional to the product of the two masses, and inversely proportional to the square of the distance between them.[2]

The equation for universal gravitation thus takes the form:

$$F = G\frac{m_1 m_2}{r^2}$$

where F is the gravitational force acting between two objects, m_1 and m_2 are the masses of the objects, r is the distance between the centers of their masses, and G is the gravitational constant.

The first test of Newton's theory of gravitation between masses in the laboratory was the Cavendish experiment conducted by the British scientist Henry Cavendish in 1798.[3] It took place 111 years after the publication of Newton's *Principia* and approximately 71 years after his death.

Newton's law of gravitation resembles Coulomb's law of electrical forces, which is used to calculate the magnitude of the electrical force arising between two charged bodies. Both are inverse-square laws, where force is inversely proportional to the square of the distance between the bodies. Coulomb's law has the product of two charges in place of the product of the masses, and the electrostatic constant in place of the gravitational constant.

Newton's law has since been superseded by Albert Einstein's theory of general relativity, but it continues to be used as an excellent approximation of the effects of gravity in most applications. Relativity is required only when there is a need for extreme accuracy, or when dealing with very strong gravitational fields, such as those found near extremely massive and dense objects, or at very close distances (such as Mercury's orbit around the Sun).

Contents

History

Early history

The relation of the distance of objects in free fall to the square of the time taken had recently been confirmed by Grimaldi and Riccioli between 1640 and 1650. They had also made a calculation of the gravitational constant by recording the oscillations of a pendulum.[4]

A modern assessment about the early history of the inverse square law is that "by the late 1670s", the assumption of an "inverse proportion between gravity and the square of distance was rather common and had been advanced by a number of different people for different reasons".[5] The same author credits Robert Hooke with a significant and seminal contribution, but treats Hooke's claim of priority on the inverse square point as irrelevant, as several individuals besides Newton and Hooke had suggested it. He points instead to the idea of "compounding the celestial motions" and the conversion of Newton's thinking away from "centrifugal" and towards "centripetal" force as Hooke's significant contributions.

Newton gave credit in his *Principia* to two people: Bullialdus (who wrote without proof that there was a force on the Earth towards the Sun), and Borelli (who wrote that all planets were attracted towards the Sun).[6][7] The main influence may have been Borelli, whose book Newton had a copy of.[8]

Plagiarism dispute

In 1686, when the first book of Newton's *Principia* was presented to the Royal Society, Robert Hooke accused Newton of plagiarism by claiming that he had taken from him the "notion" of "the rule of the decrease of Gravity, being reciprocally as the squares of the distances from the Center". At the same time (according to Edmond Halley's contemporary report) Hooke agreed that "the Demonstration of the Curves generated thereby" was wholly Newton's.[9]

In this way, the question arose as to what, if anything, Newton owed to Hooke. This is a subject extensively discussed since that time and on which some points, outlined below, continue to excite controversy.

Hooke's work and claims

Robert Hooke published his ideas about the "System of the World" in the 1660s, when he read to the Royal Society on March 21, 1666, a paper "concerning the inflection of a direct motion into a curve by a supervening attractive principle", and he published them again in somewhat developed form in 1674, as an addition to "An Attempt to Prove the Motion of the Earth from Observations".[10] Hooke announced in 1674 that he planned to "explain a System of the World differing in many particulars from any yet known", based on three "Suppositions": that "all Celestial Bodies whatsoever, have an attraction or gravitating power towards their own Centers" and "they do also attract all the other Celestial Bodies that are within the sphere of their activity";[11] that "all bodies whatsoever that are put into a direct and simple motion, will so continue to move forward in a straight line, till they are by some other effectual powers deflected and bent..."; and that "these attractive powers are so much the more powerful in operating, by how much the nearer the body wrought upon is to their own Centers". Thus Hooke clearly postulated mutual attractions between the Sun and planets, in a way that increased with nearness to the attracting body, together with a principle of linear inertia.

Hooke's statements up to 1674 made no mention, however, that an inverse square law applies or might apply to these attractions. Hooke's gravitation was also not yet universal, though it approached universality more closely than previous hypotheses.[12] He also did not provide accompanying evidence or mathematical demonstration. On the latter two aspects, Hooke himself stated in 1674: "Now what these several degrees [of attraction] are I have not yet experimentally verified"; and as to his whole proposal: "This I only hint at present", "having my self many other things in hand which I would first compleat, and therefore cannot so well attend it" (i.e. "prosecuting this Inquiry").[10] It was later on, in writing on 6 January 1679|80[13] to Newton, that Hooke communicated his "supposition ... that the Attraction always is in a duplicate proportion to the Distance from the Center Reciprocall, and Consequently that the Velocity will be in a subduplicate proportion to the Attraction and Consequently as Kepler Supposes Reciprocall to the Distance."[14] (The inference about the velocity was incorrect.)[15]

Hooke's correspondence with Newton during 1679–1680 not only mentioned this inverse square supposition for the decline of attraction with increasing distance, but also, in Hooke's opening letter to Newton, of 24 November 1679, an approach of "compounding the celestial motions of the planets of a direct motion by the tangent & an attractive motion towards the central body".[16]

Newton's work and claims

Newton, faced in May 1686 with Hooke's claim on the inverse square law, denied that Hooke was to be credited as author of the idea. Among the reasons, Newton recalled that the idea had been discussed with Sir Christopher Wren previous to Hooke's 1679 letter.[17] Newton also pointed out and acknowledged prior work of others,[18] including Bullialdus,[6] (who suggested, but without demonstration, that there was an attractive force from the

Sun in the inverse square proportion to the distance), and Borelli[7] (who suggested, also without demonstration, that there was a centrifugal tendency in counterbalance with a gravitational attraction towards the Sun so as to make the planets move in ellipses). D T Whiteside has described the contribution t Newton's thinking that came from Borelli's book, a copy of which was in Newton's library at his death.[8]

Newton further defended his work by saying that had he first heard of the inverse square proportion from Hooke, he would still have some rights to it in view of his demonstrations of its accuracy. Hooke, without evidence in favor of the supposition, could only guess that the inverse square law was approximately valid at great distances from the center. According to Newton, while the 'Principia' was still at pre-publication stage, there were so many a-priori reasons to doubt the accuracy of the inverse-square law (especially close to an attracting sphere) that "without my (Newton's) Demonstrations, to which Mr Hooke is yet a stranger, it cannot believed by a judicious Philosopher to be any where accurate."[19]

This remark refers among other things to Newton's finding, supported by mathematical demonstration, that if the inverse square law applies to tiny particles, then even a large spherically symmetrical mass also attracts masses external to its surface, even close up, exactly as if all its own mass were concentrated at its center. Thus Newton gave a justification, otherwise lacking, for applying the inverse square law to large spherical planetary masses as if they were tiny particles.[20] In addition, Newton had formulated, in Propositions 43-45 of Book 1[21] and associated sections of Book 3, a sensitive test of the accuracy of the inverse square law, in which he showed that only where the law of force is calculated as the inverse square of the distance will the directions of orientation of the planets' orbital ellipses stay constant as they are observed to do apart from small effects attributable to inter-planetary perturbations.

In regard to evidence that still survives of the earlier history, manuscripts written by Newton in the 1660s show that Newton himself had, by 1669, arrived at proofs that in a circular case of planetary motion, "endeavour to recede" (what was later called centrifugal force) had an inverse-square relation with distance from the center.[22] After his 1679-1680 correspondence with Hooke, Newton adopted the language of inward or centripetal force. According to Newton scholar J. Bruce Brackenridge, although much has been made of the change in language and difference of point of view, as between centrifugal or centripetal forces, the actual computations and proofs remained the same either way. They also involved the combination of tangential and radial displacements, which Newton was making in the 1660s. The lesson offered by Hooke to Newton here, although significant, was one of perspective and did not change the analysis.[23] This background shows there was basis for Newton to deny deriving the inverse square law from Hooke.

Newton's acknowledgment

On the other hand, Newton did accept and acknowledge, in all editions of the *Principia*, that Hooke (but not exclusively Hooke) had separately appreciated the inverse square law in the solar system. Newton acknowledged Wren, Hooke and Halley in this connection in the Scholium to Proposition 4 in Book 1.[24] Newton also acknowledged to Halley that his correspondence with Hooke in 1679-80 had reawakened his dormant interest in astronomical matters, but that did not mean, according to Newton, that Hooke had told Newton anything new or original: "yet am I not beholden to him for any light into that business but only for the diversion he gave me from my other studies to think on these things & for his dogmaticalness in writing as if he had found the motion in the Ellipsis, which inclined me to try it ..."[18]

Modern priority controversy

Since the time of Newton and Hooke, scholarly discussion has also touched on the question of whether Hooke's 1679 mention of 'compounding the motions' provided Newton with something new and valuable, even though that was not a claim actually voiced by Hooke at the time. As described above, Newton's manuscripts of the 1660s do show him actually combining tangential motion with the effects of radially directed force or endeavour, for example in his derivation of the inverse square relation for the circular case. They also show Newton clearly expressing the concept of linear inertia—for which he was indebted to Descartes' work, published in 1644 (as Hooke probably was).[25] These matters do not appear to have been learned by Newton from Hooke.

Nevertheless, a number of authors have had more to say about what Newton gained from Hooke and some aspects remain controversial.[5] The fact that most of Hooke's private papers had been destroyed or have disappeared does not help to establish the truth.

Newton's role in relation to the inverse square law was not as it has sometimes been represented. He did not claim to think it up as a bare idea. What Newton did was to show how the inverse-square law of attraction had many necessary mathematical connections with observable features of the motions of bodies in the solar system; and that they were related in such a way that the observational evidence and the mathematical demonstrations, taken together, gave reason to believe that the inverse square law was not just approximately true but exactly true (to the accuracy achievable in Newton's time and for about two centuries afterwards – and with some loose ends of points that could not yet be certainly examined, where the implications of the theory had not yet been adequately identified or calculated).[26][27]

About thirty years after Newton's death in 1727, Alexis Clairaut, a mathematical astronomer eminent in his own right in the field of gravitational studies, wrote after reviewing what Hooke published, that "One must not think that this idea ... of Hooke diminishes Newton's glory"; and that "the example of Hooke" serves "to show what a distance there is between a truth that is glimpsed and a truth that is demonstrated".[28][29]

Modern form

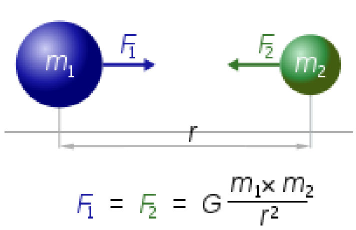
In modern language, the law states the following:

Every point mass attracts every single other point mass by a force acting along the line intersecting both points. The force is proportional to the product of the two masses and inversely proportional to the square of the distance between them:[2]

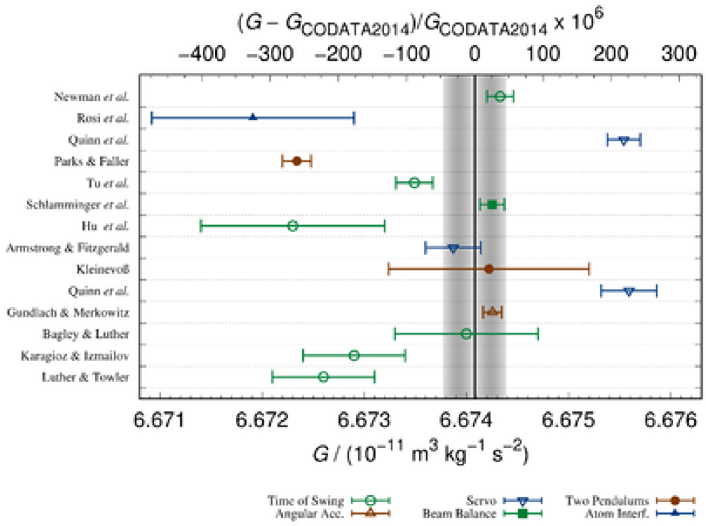
$$F = G\frac{m_1 m_2}{r^2}$$

where:

- F is the force between the masses;
- G is the gravitational constant (6.674×10^{-11} N · (m/kg)^2);
- m_1 is the first mass;
- m_2 is the second mass;
- r is the distance between the centers of the masses.



Assuming SI units, F is measured in newtons (N), m_1 and m_2 in kilograms (kg), r in meters (m), and the constant G is approximately equal to 6.674×10^{-11} N m^2 kg^{-2}.[30] The value of the constant G was first accurately determined from the results of the Cavendish experiment conducted by the British scientist Henry Cavendish in 1798, although Cavendish did not himself calculate a numerical value for G.[3] This experiment was also the first test of Newton's theory of gravitation between masses in the laboratory. It took place 111 years after the publication of Newton's *Principia* and 71 years after Newton's death, so none of Newton's calculations could use the value of G; instead he could only calculate a force relative to another force.

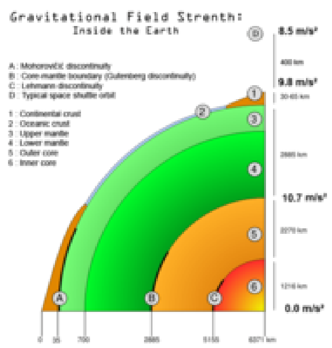


Error plot showing experimental values for big G.

Bodies with spatial extent

If the bodies in question have spatial extent (as opposed to being point masses), then the gravitational force between them is calculated by summing the contributions of the notional point masses which constitute the bodies. In the limit, as the component point masses become "infinitely small", this entails integrating the force (in vector form, see below) over the extents of the two bodies.

In this way, it can be shown that an object with a spherically-symmetric distribution of mass exerts the same gravitational attraction on external bodies as if all the object's mass were concentrated at a point at its centre.[2] (This is not generally true for non-spherically-symmetrical bodies.)



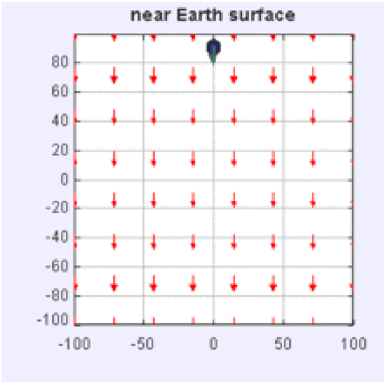
Gravitational field strength within the Earth

For points *inside* a spherically-symmetric distribution of matter, Newton's Shell theorem can be used to find the gravitational force. The theorem tells us how different parts of the mass distribution affect the gravitational force measured at a point located a distance r_0 from the center of the mass distribution:[31]

- The portion of the mass that is located at radii $r < r_0$ causes the same force at r_0 as if all of the mass enclosed within a sphere of radius r_0 was concentrated at the center of the mass distribution (as noted above).
- The portion of the mass that is located at radii $r > r_0$ exerts *no net* gravitational force at the distance r_0 from the center. That is, the individual gravitational forces exerted by the elements of the sphere out there, on the point at r_0, cancel each other out.

As a consequence, for example, within a shell of uniform thickness and density there is *no net* gravitational acceleration anywhere within the hollow sphere.

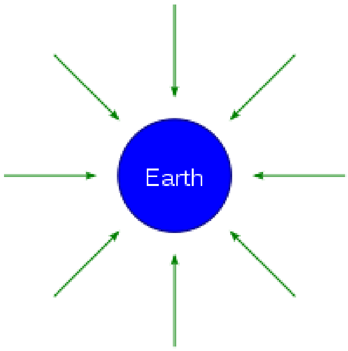
Furthermore, inside a uniform sphere the gravity increases linearly with the distance from the center; the increase due to the additional mass is 1.5 times the decrease due to the larger distance from the center. Thus, if a spherically symmetric body has a uniform core and a uniform mantle with a density that is less than 2/3 of that of the core, then the gravity initially decreases outwardly beyond the boundary, and if the sphere is large enough, further outward the gravity increases again, and eventually it exceeds the gravity at the core/mantle boundary. The gravity of the Earth may be highest at the core/mantle boundary.



Gravity field near the surface of the Earth – an object is shown accelerating toward the surface

Vector form

Newton's law of universal gravitation can be written as a vector equation to account for the direction of the gravitational force as well as its magnitude. In this formula, quantities in bold represent vectors.



Gravity field surrounding Earth from a macroscopic perspective.

$$\mathbf{F}_{21} = -G\frac{m_1 m_2}{|\mathbf{r}_{12}|^2}\,\hat{\mathbf{r}}_{12}$$

where

$\mathbf{F}_{21}$ is the force applied on object 2 exerted by object 1,
G is the gravitational constant,
m_1 and m_2 are respectively the masses of objects 1 and 2,
$|\mathbf{r}_{12}| = |\mathbf{r}_2 - \mathbf{r}_1|$ is the distance between objects 1 and 2, and
$\hat{\mathbf{r}}_{12} \stackrel{\text{def}}{=} \frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|}$ is the unit vector from object 1 to 2.

It can be seen that the vector form of the equation is the same as the scalar form given earlier, except that **F** is now a vector quantity, and the right hand side is multiplied by the appropriate unit vector. Also, it can be seen that $\mathbf{F}_{12} = -\mathbf{F}_{21}$.

Gravitational field

The **gravitational field** is a vector field that describes the gravitational force which would be applied on an object in any given point in space, per unit mass. It is actually equal to the gravitational acceleration at that point.

It is a generalisation of the vector form, which becomes particularly useful if more than 2 objects are involved (such as a rocket between the Earth and the Moon). For 2 objects (e.g. object 2 is a rocket, object 1 the Earth), we simply write $\mathbf{r}$ instead of $\mathbf{r}_{12}$ and m instead of m_2 and define the gravitational field $\mathbf{g}(\mathbf{r})$ as:

$$\mathbf{g}(\mathbf{r}) = -G\frac{m_1}{|\mathbf{r}|^2}\,\hat{\mathbf{r}}$$

so that we can write:

$$\mathbf{F}(\mathbf{r}) = m\mathbf{g}(\mathbf{r}).$$

This formulation is dependent on the objects causing the field. The field has units of acceleration; in SI, this is m/s^2.

Gravitational fields are also conservative; that is, the work done by gravity from one position to another is path-independent. This has the consequence that there exists a gravitational potential field $V(\mathbf{r})$ such that

$$\mathbf{g}(\mathbf{r}) = -\nabla V(\mathbf{r}).$$

If m_1 is a point mass or the mass of a sphere with homogeneous mass distribution, the force field $\mathbf{g}(\mathbf{r})$ outside the sphere is isotropic, i.e., depends only on the distance r from the center of the sphere. In that case

$$V(r) = -G\frac{m_1}{r}.$$

the gravitational field is on, inside and outside of symmetric masses.

As per Gauss Law, field in a symmetric body can be found by the mathematical equation:

$$\oiint_{\partial V} \mathbf{g(r)} \cdot d\mathbf{A} = -4\pi G M_{\text{enc}},$$

where ∂V is a closed surface and M_{enc} is the mass enclosed by the surface.

Hence, for a hollow sphere of radius R and total mass M,

$$|\mathbf{g(r)}| = \begin{cases} 0, & \text{if } r < R \\ \dfrac{GM}{r^2}, & \text{if } r \geq R \end{cases}$$

For a uniform solid sphere of radius R and total mass M,

$$|\mathbf{g(r)}| = \begin{cases} \dfrac{GMr}{R^3}, & \text{if } r < R \\ \dfrac{GM}{r^2}, & \text{if } r \geq R \end{cases}$$

Problematic aspects

Newton's description of gravity is sufficiently accurate for many practical purposes and is therefore widely used. Deviations from it are small when the dimensionless quantities φ/c^2 and $(v/c)^2$ are both much less than one, where φ is the gravitational potential, v is the velocity of the objects being studied, and c is the speed of light.[32] For example, Newtonian gravity provides an accurate description of the Earth/Sun system, since

$$\frac{\Phi}{c^2} = \frac{GM_{\text{sun}}}{r_{\text{orbit}} c^2} \sim 10^{-8}, \quad \left(\frac{v_{\text{Earth}}}{c}\right)^2 = \left(\frac{2\pi r_{\text{orbit}}}{(1\text{ yr})c}\right)^2 \sim 10^{-8}$$

where r_{orbit} is the radius of the Earth's orbit around the Sun.

In situations where either dimensionless parameter is large, then general relativity must be used to describe the system. General relativity reduces to Newtonian gravity in the limit of small potential and low velocities, so Newton's law of gravitation is often said to be the low-gravity limit of general relativity.

Theoretical concerns with Newton's expression

- There is no immediate prospect of identifying the mediator of gravity. Attempts by physicists to identify the relationship between the gravitational force and other known fundamental forces are not yet resolved, although considerable headway has been made over the last 50 years (See: Theory of everything and Standard Model). Newton himself felt that the concept of an inexplicable *action at a distance* was unsatisfactory (see "Newton's reservations" below), but that there was nothing more that he could do at the time.
- Newton's theory of gravitation requires that the gravitational force be transmitted instantaneously. Given the classical assumptions of the nature of space and time before the development of General Relativity, a significant propagation delay in gravity leads to unstable planetary and stellar orbits.

Observations conflicting with Newton's formula

- Newton's Theory does not fully explain the precession of the perihelion of the orbits of the planets, especially of planet Mercury, which was detected long after the life of Newton.[33] There is a 43 arcsecond per century discrepancy between the Newtonian calculation, which arises only from the gravitational attractions from the other planets, and the observed precession, made with advanced telescopes during the 19th century
- The predicted angular deflection of light rays by gravity that is calculated by using Newton's Theory is only one-half of the deflection that is actually observed by astronomers. Calculations using General Relativity are in much closer agreement with the astronomical

observations.

- In spiral galaxies, the orbiting of stars around their centers seems to strongly disobey Newton's law of universal gravitation. Astrophysicists, however, explain this spectacular phenomenon in the framework of Newton's laws, with the presence of large amounts of dark matter.

Newton's reservations

While Newton was able to formulate his law of gravity in his monumental work, he was deeply uncomfortable with the notion of "action at a distance" that his equations implied. In 1692, in his third letter to Bentley, he wrote: *"That one body may act upon another at a distance through a vacuum without the mediation of anything else, by and through which their action and force may be conveyed from one another, is to me so great an absurdity that, I believe, no man who has in philosophic matters a competent faculty of thinking could ever fall into it."*

He never, in his words, "assigned the cause of this power". In all other cases, he used the phenomenon of motion to explain the origin of various forces acting on bodies, but in the case of gravity, he was unable to experimentally identify the motion that produces the force of gravity (although he invented two mechanical hypotheses in 1675 and 1717). Moreover, he refused to even offer a hypothesis as to the cause of this force on grounds that to do so was contrary to sound science. He lamented that "philosophers have hitherto attempted the search of nature in vain" for the source of the gravitational force, as he was convinced "by many reasons" that there were "causes hitherto unknown" that were fundamental to all the "phenomena of nature". These fundamental phenomena are still under investigation and, though hypotheses abound, the definitive answer has yet to be found. And in Newton's 1713 *General Scholium* in the second edition of *Principia*: *"I have not yet been able to discover the cause of these properties of gravity from phenomena and I feign no hypotheses... It is enough that gravity does really exist and acts according to the laws I have explained, and that it abundantly serves to account for all the motions of celestial bodies."*[34]

Einstein's solution

These objections were explained by Einstein's theory of general relativity, in which gravitation is an attribute of curved spacetime instead of being due to a force propagated between bodies. In Einstein's theory, energy and momentum distort spacetime in their vicinity, and other particles move in trajectories determined by the geometry of spacetime. This allowed a description of the motions of light and mass that was consistent with all available observations. In general relativity, the gravitational force is a fictitious force due to the curvature of spacetime, because the gravitational acceleration of a body in free fall is due to its world line being a geodesic of spacetime.

Extensions

Newton was the first to consider in his *Principia* an extended expression of his law of gravity including an inverse-cube term of the form

$$F = G\frac{m_1 m_2}{r^2} + B\frac{m_1 m_2}{r^3}\ ,\ B \text{ a constant}$$

attempting to explain the Moon's apsidal motion. Other extensions were proposed by Laplace (around 1790) and Decombes (1913):[35]

$$F(r) = k\frac{m_1 m_2}{r^2}\exp(-\alpha r) \text{ (Laplace)}$$

$$F(r) = k\frac{m_1 m_2}{r^2}\left(1 + \frac{\alpha}{r^3}\right) \text{ (Decombes)}$$

In recent years, quests for non-inverse square terms in the law of gravity have been carried out by neutron interferometry.[36]

Solutions of Newton's law of universal gravitation

The *n*-body problem is an ancient, classical problem[37] of predicting the individual motions of a group of celestial objects interacting with each other gravitationally. Solving this problem — from the time of the Greeks and on — has been motivated by the desire to understand the motions of the Sun, planets and the visible stars. In the 20th century, understanding the dynamics of globular cluster star systems became an important *n*-body problem too.[38] The *n*-body problem in general relativity is considerably more difficult to solve.

The classical physical problem can be informally stated as: *given the quasi-steady orbital properties* (*instantaneous position, velocity and time*)[39] *of a group of celestial bodies, predict their interactive forces; and consequently, predict their true orbital motions for all future times.*[40]

The two-body problem has been completely solved, as has the restricted three-body problem.[41]

See also

- Bentley's paradox

- Gauss's law for gravity
- Jordan and Einstein frames
- Kepler orbit
- Newton's cannonball
- Newton's laws of motion
- Static forces and virtual-particle exchange

Notes

1. It was shown separately that large, spherically symmetrical masses attract and are attracted as if all their mass were concentrated at their centers.

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16. Page 297 in H W Turnbull (ed.), Correspondence of Isaac Newton, Vol 2 (1676-1687), (Cambridge University Press, 1960), document #235, 24 November 1679.
17. Page 433 in H W Turnbull (ed.), Correspondence of Isaac Newton, Vol 2 (1676-1687), (Cambridge University Press, 1960), document #286, 27 May 1686.
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19. Page 436, Correspondence, Vol.2, already cited.
20. Propositions 70 to 75 in Book 1, for example in the 1729 English translation of the *Principia*, start at page 263 (https://books.google.com/books?id=Tm0FAAAAQAAJ&pg=PA263#v=onepage&q=&f=false)
21. Propositions 43 to 45 in Book 1, in the 1729 English translation of the *Principia*, start at page 177 (https://books.google.com/books?id=Tm0FAAAAQAAJ&pg=PA177#v=onepage&q=&f=false)
22. D T Whiteside, "The pre-history of the 'Principia' from 1664 to 1686", Notes and Records of the Royal Society of London, 45 (1991), pages 11-61; especially at 13-20.
23. See J. Bruce Brackenridge, "The key to Newton's dynamics: the Kepler problem and the Principia", (University of California Press, 1995), especially at pages 20-21 (https://books.google.com/books?id=ovOTK7X_mMkC&pg=PA20#v=onepage&q=&f=false)

24. See for example the 1729 English translation of the *Principia*, at page 66 (https://books.google.com/books?id=Tm0FAAAAQAAJ&pg=PA66#v=onepage&q=&f=false)
25. See page 10 in D T Whiteside, "Before the Principia: the maturing of Newton's thoughts on dynamical astronomy, 1664-1684", Journal for the History of Astronomy i (1970), pages 5-19.
26. See for example the results of Propositions 43-45 and 70-75 in Book 1, cited above.
27. See also G E Smith, in Stanford Encyclopedia of Philosophy, "Newton's Philosophiae Naturalis Principia Mathematica" (http://plato.stanford.edu/entries/newton-principia/)
28. The second extract is quoted and translated in W.W. Rouse Ball, "An Essay on Newton's 'Principia'" (London and New York: Macmillan, 1893), at page 69.
29. The original statements by Clairaut (in French) are found (with orthography here as in the original) in "Explication abregée du systême du monde, et explication des principaux phénomenes astronomiques tirée des Principes de M. Newton" (1759), at Introduction (section IX), page 6: "Il ne faut pas croire que cette idée ... de Hook diminue la gloire de M. Newton", and "L'exemple de Hook" [serve] "à faire voir quelle distance il y a entre une vérité entrevue & une vérité démontrée".
30. Mohr, Peter J.; Taylor, Barry N.; Newell, David B. (2008). "CODATA Recommended Values of the Fundamental Physical Constants: 2006" (https://web.archive.org/web/20171001121533/https://www.nist.gov/sites/default/files/documents/pml/div684/fcdc/rmp2006-2.pdf) (PDF). *Reviews of Modern Physics* **80** (2): 633–730. arXiv:0801.0028 (https://arxiv.org/abs/0801.0028). Bibcode:2008RvMP...80..633M (http://adsabs.harvard.edu/abs/2008RvMP...80..633M). doi:10.1103/RevModPhys.80.633 (https://doi.org/10.1103%2FRevModPhys.80.633). Archived from the original (https://www.nist.gov/pml/div684/fcdc/upload/rmp2006-2.pdf) (PDF) on 2017-10-01. Direct link to value (http://www.physics.nist.gov/cgi-bin/cuu/Value?bg)..
31. Equilibrium State (http://farside.ph.utexas.edu/teaching/336k/lectures/node109.html)
32. Misner, Charles W.; Thorne, Kip S.; Wheeler, John Archibald (1973). *Gravitation*. New York: W. H.Freeman and Company. ISBN 978-0-7167-0344-0 Page 1049.
33. - Max Born (1924), *Einstein's Theory of Relativity* (The 1962 Dover edition, page 348 lists a table documenting the observed and calculated values for the precession of the perihelion of Mercury, Venus, and the Earth.)
34. - *The Construction of Modern Science: Mechanisms and Mechanics*, by Richard S. Westfall. Cambridge University Press. 1978
35. Has, Ioan; Miclaus, Simona; Has, Aurelian (December 2008). "Analysis of a possible correlation between electrical and gravitational forces". *Physics Essays*. **21** (4): 303–312. doi:10.4006/1.3038751 (https://doi.org/10.4006%2F1.3038751).
36. Greene, Geoffrey L.; Gudkov, Vladimir (2007). "Neutron interferometric method to provide improved constraints on non-Newtonian gravity at the nanometer scale". *Physical Review C*. **75** (1): 015501. arXiv:hep-ph/0608346 (https://arxiv.org/abs/hep-ph/0608346). Bibcode:2007PhRvC..75a5501G (http://adsabs.harvard.edu/abs/2007PhRvC..75a5501G). doi:10.1103/PhysRevC.75.015501 (https://doi.org/10.1103%2FPhysRevC.75.015501).
37. Leimanis and Minorsky: Our interest is with Leimanis, who first discusses some history about the *n*-body problem, especially Ms. Kovalevskaya's ~1868-1888, twenty-year complex-variables approach, failure; **Section 1: The Dynamics of Rigid Bodies and Mathematical Exterior Ballistics** (Chapter 1, *the motion of a rigid body about a fixed point* (**Euler** and **Poisson** *equations*); Chapter 2, *Mathematical Exterior Ballistics*), good precursor background to the *n*-body problem; **Section 2: Celestial Mechanics** (Chapter 1, *The Uniformization of the Three-body Problem* (Restricted Three-body Problem); Chapter 2, *Capture in the Three-Body Problem*; Chapter 3, *Generalized n-body Problem*).
38. See References sited for Heggie and Hut. This Wikipedia page has made their approach obsolete.
39. *Quasi-steady* loads refers to the instantaneous inertial loads generated by instantaneous angular velocities and accelerations, as well as translational accelerations (9 variables). It is as though one took a photograph, which also recorded the instantaneous position and properties of motion. In contrast, a *steady-state* condition refers to a system's state being invariant to time; otherwise, the first derivatives and all higher derivatives are zero.
40. R. M. Rosenberg states the *n*-body problem similarly (see References): *Each particle in a system of a finite number of particles is subjected to a Newtonian gravitational attraction from all the other particles, and to no other forces. If the initial state of the system is given, how will the particles move?* Rosenberg failed to realize, like everyone else, that it is necessary to determine the forces *first* before the motions can be determined.
41. A general, classical solution in terms of first integrals is known to be impossible. An exact theoretical solution for arbitrary *n* can be approximated via Taylor series, but in practice such an infinite series must be truncated, giving at best only an approximate solution; and an approach now obsolete. In addition, the *n*-body problem may be solved using numerical integration, but these, too, are approximate solutions; and again obsolete. See Sverre J. Aarseth's book **Gravitational N-body Simulations** listed in the References.

External links

- Feather and Hammer Drop on Moon on YouTube
- Newton's Law of Universal Gravitation Javascript calculator