

Directions: Please answer all items on this homework. You must show all your work. Unless otherwise stated, please simplify your answer.

1. Suppose you sold a product y at an output price of \$30. Suppose that the production of y uses an input x that costs \$240 per unit. Answer the following questions based on the following production function: $y = f(x) = 38x - x^2$. Please show how you found your answer. **(45 Points)**
 - a. Set-up the constrained profit maximization problem. What is the Lagrangean and first order condition for this problem?
 - b. Solve the first order conditions to find the optimal input level, optimal output level, and profit.
 - c. Verify that your optimal input in part b is truly the optimal input by using $MVP = MIC$.
 - d. Verify that your optimal input in part b is truly the optimal input by using $MPP = w/p$.
 - e. Verify that your optimal output in part b is truly the optimal output by using $MR = MC$.
 - f. What stage is your optimal solution in?
 - g. Set-up the unconstrained profit maximization problem with respect to input and find where the optimal input occurs.
 - h. Set-up the unconstrained profit maximization problem with respect to output and find where the optimal output occurs.
 - i. How much profit would you have lost if you produced at the maximum revenue point?
2. Suppose you sold a product y at an output price of \$5. Suppose that the production of y uses two inputs x_1 and x_2 . The cost of input 1 is \$120 while the cost of input 2 is \$80 per unit. Answer the following questions based on the following production function: $y = f(x_1, x_2) = 42x_1 - 3x_1^2 + 32x_2 - 4x_2^2$. Please show how you found your answer. **(25 Points)**
 - a. Set-up the constrained profit maximization problem.
 - b. What is the Lagrangean and first order condition for this problem?
 - c. Solve the first order conditions to find the optimal input level, optimal output level, and profit.
 - d. Solve for the optimal inputs in part a by using $MPP_{x_1} = w_1/p$ and $MPP_{x_2} = w_2/p$; this should verify your answer in part c is correct.
 - e. How much profit would you have lost if you produced at the maximum revenue point?

3. Suppose you sold a product y at an output price of \$8. Suppose that the production of y uses two inputs x_1 and x_2 . The cost of input 1 is \$4 while the cost of input 2 is \$256 per unit. Answer the following questions based on the following production function: $y = f(x_1, x_2) = 40x_1^{2/5}x_2^{1/5}$. Please show how you found your answer. **(25 Points)**
- What are the returns to scale of this production function?
 - Set-up the constrained profit maximization problem.
 - What is the Lagrangean and first order condition for this problem?
 - Solve the first order conditions to find the optimal input level, optimal output level, and profit.
 - Solve for the optimal inputs in part a by using $MPP_{x_1} = w_1/p$ and $MPP_{x_2} = w_2/p$; this should verify your answer in part c is correct.
4. Suppose you have 20,000 acres of land to allocate to corn and soybeans. The production function for corn is $Y_1 = 20x_1^{1/4}$, where Y_1 is the amount of bushels of corn and x_1 is the amount of land used for corn. You also know that the production function for soybeans is $Y_2 = 40x_2^{1/4}$, where Y_2 is the amount of bushels of soybeans and x_2 is the amount of land used for soybeans. Assume that the price of corn is 320 and the price of soybeans is 160. Also assume the input cost is 1.6. **(45 Points)**
- Set-up the maximization problem.
 - Set-up the Lagrangean and give the first order conditions.
 - Please find the production possibility frontier (PPF) using soybeans as the dependent variable and corn as the independent variable?
 - What is the marginal rate of product transformation (MRPT) for the PPF?
 - What is the optimal output given the price ratio?
 - Solve for the optimal input use by using the MVP's.
 - How much revenue and profit are being brought in at the optimal outputs and input allocation?
 - Please sketch the solution on a graph. (Note, all the numbers on the graph may not come out to be integer numbers.)
 - What is the MRPT at $Y_1 = 200$? Please explain what this means in an economic sense.