

**Directions:** Please answer all items on this homework. You must show all your work. Unless otherwise stated, please simplify your answer.

1. Answer the following questions based on the following production function:  
 $y = f(x) = -3x^3 + 24x^2 + 60x$ . Also suppose that the input price for  $x$  is \$540. Please show how you found your answer. **(40 Points)**
  - a. What input levels give positive output levels that make economic sense?
  - b. What is the marginal physical product (MPP) function for this production function?  
What is the marginal cost function as a function of inputs?
  - c. What is the average physical product (APP) function for this production function? What is the average variable cost function as a function of inputs?
  - d. What input range gives marginal cost greater than average variable cost?
  - e. What input gives marginal cost equal to average variable cost?
  - f. What input range gives marginal cost less than average variable cost?
  - g. Assuming that this producer would like to produce at the lowest possible average variable cost while producing a positive output, what level of input and output would this person produce at? Please explain how you found this answer?
  - h. What is the average variable cost at your optimal input-output combination?
  
2. Answer the following questions based on the following production function:  
 $y = f(x) = 30x - x^2$ . Assume that the cost of the input is \$20. Please show how you found your answer. **(40 Points)**
  - a. Assuming that this producer is a cost minimizer, what is the person's cost minimization problem.
  - b. What are the Lagrangean and the first order conditions for this problem?
  - c. Given the first order conditions, what is the optimal input level for any given output level  $y$ ?
  - d. Suppose you wanted to produce 125 units, how much input would you use? What is the minimum cost of producing 125 units?
  - e. Starting from the original production function, derive the cost function as a function of output.
  - f. Using the information in part e, what is the marginal cost of the above cost function?
  - g. Demonstrate that the marginal cost function you derived in part f is equivalent to what  $\lambda$  is from your first order conditions from part b.
  - h. What input or input range gives:
    - a. marginal cost greater than average variable cost,
    - b. marginal cost equal to average variable cost, and
    - c. marginal cost less than average variable cost?

3. Answer the following questions based on the following production function:  
 $y = f(x_1, x_2) = 3x_1^{1/2}x_2^{1/3}$ . Also, assume that the price of input 1 is \$9 and the price of input 2 is \$60. Please show how you found your answer. **(55 Points)**
- Assuming that this producer is a cost minimizer, what is the person's cost minimization problem.
  - What are the Lagrangean and the first order conditions for this problem?
  - Solve part b for the optimal input levels as a function of  $y$ .
  - If you wanted to produce 3,000 units, how much of input 1 and input 2 would you use?
  - What is the minimum cost to produce 3,000 units?
  - Calculate the isoquant for the production function above and find the MRTS from the isoquant.
  - Solve this problem by setting MRTS equal to the slope of the iso-cost line and solving for  $x_1$  and  $x_2$ ; this should verify that your answers from part d are correct?
  - Calculate the marginal physical products for the production function above.
  - Solve this problem by using the negative ratio of the marginal physical products set equal to the slope of the iso-cost line and solving for  $x_1$  and  $x_2$ ; this should verify that your answers from part d are correct?
  - Would the optimal decision in part d change if input one increased to \$18 and the price of input 2 increased to \$120? Please explain.
  - If the price of input one dropped to \$3, what would the price of input 2 need to drop to maintain the same optimal inputs found in part d? Please explain.
4. Suppose your goal this year is to produce 30,000 bushels of organic corn. Suppose your production technology has the following relationship for producing bushels of corn  $Q = f(L, T) = 30L^{1/3}T^{1/2}$  where  $Q$  is the number of bushels of corn you produce,  $L$  is the number of labor hours you utilize, and  $T$  is the number of tractor hours you utilize. You know that the cost per hour of labor is \$600 and the cost per hour of tractor time is \$90. Assume that you are a cost minimizing producer. **(50 points)**
- Set-up the cost minimization problem.
  - Solve this problem using the Lagrangean method. **(10 points)**
  - Solve this problem by deriving the isoquant and using the fact that  $|MRTS|$  is equal to the input price ratio. **(10 points)**
  - Solve this problem using the ratios of the marginal physical products equal to the price ratio. **(10 points)**
  - How much money will you need to ask from your banker to achieve your goal?
  - Please carefully sketch a graph of this solution. Include the isoquant and iso-cost line as well as all relevant information. **(10 points)**