

Directions: Please answer all items on this homework. You must show all your work. Unless otherwise stated, please simplify your answer.

1. Answer the following questions based on the following production function:
 $y = f(x) = -7x^3 + 210x^2$. Please show how you found your answer. **(70 Points)**
 - a. What is the marginal physical product (MPP) function for this production function?
 - b. In terms of inputs, show where the MPP function is at a maximum.
 - c. Demonstrate that MPP is at a maximum by using derivatives.
 - d. What is the highest achievable MPP?
 - e. What is the average physical product (APP) function for this production function?
 - f. In terms of inputs, show where the APP function is at a maximum.
 - g. Demonstrate that APP is at a maximum by using derivatives.
 - h. What is the highest achievable APP?
 - i. Where is the maximum point on the production function?
 - j. Demonstrate that the maximum point that you found for the production function is truly a maximum.
 - k. What is the highest output achievable?
 - l. Where does Stage I of the production function turn into Stage II of the production function?
 - m. Where does Stage II of the production function turn into Stage III of the production function?
 - n. Where does the Law of Diminishing Marginal Returns take effect?
2. Answer the following questions based on the following production function:
 $y = f(x_1, x_2) = -x_1^2 + 6x_1 - x_2^2 + 16x_2$. Please show how you found your answer. **(60 Points)**
 - a. What is the marginal physical product (MPP_{x_1}) function for this production function with respect to input 1?
 - b. What is the marginal physical product (MPP_{x_2}) function for this production function with respect to input 2?
 - c. What is the extrema point to this production function? (Note, the extrema point is made up of x_1 and x_2 together.)
 - d. What is the maximum you can achieve for output?
 - e. Find the isoquant for any given output level? (Note: make sure you find the one that makes economic sense.)
 - f. If output is equal to 68 and input 1 is equal to 1, how much input 2 do you need?
 - g. Calculate the marginal rate of technical substitution (MRTS) using the fact that MRTS is equal to the negative ratio of the MPP's.

- h. Calculate the marginal rate of technical substitution (MRTS) in terms of inputs by taking the derivative of the isoquant you found in part d.
 - i. Given the information in part f and h, what is the MRTS?
 - j. Please explain what part i means in an economic sense?
 - k. Assuming that x_1 is fixed at an input level of 2, what is the highest achievable output?
 - l. Demonstrate that x_1 in part k is the highest achievable output by using derivatives.
3. Answer the following questions based on the following production function:
 $y = f(x_1, x_2) = 60x_1^{1/4}x_2^{1/2}$. Please show how you found your answer. **(50 Points)**
- a. What is the marginal physical product (MPP_{x_1}) function for this production function with respect to input 1?
 - b. What is the marginal physical product (MPP_{x_2}) function for this production function with respect to input 2?
 - c. Find the isoquant for any given output level.
 - d. If output is equal to 240 and input 1 is equal to 16, how much input 2 do you need?
 - e. Calculate the marginal rate of technical substitution (MRTS) using the fact that MRTS is equal to the negative ratio of the MPP's.
 - f. Calculate the marginal rate of technical substitution (MRTS) in terms of inputs by taking the derivative of the isoquant you found in part b.
 - g. Given the information in parts d and f, what is the MRTS? Please explain what this means in an economic sense?
 - h. What are the returns to scale for this production function?
 - i. Suppose you had a new production function that is $y = f(x_1, x_2, x_3) = 20x_1^{1/4}x_2^{1/2}x_3^{1/4}$, what are the returns to scale of this production function?
 - j. Assuming that x_3 is the dependent variable, find the isoquant for any given output level for the production function in part h.
4. Suppose you have 100 acres of land to allocate to corn and soybeans. The production function for corn is $Y_1 = 50x_1^{1/2}$, where Y_1 is the amount of bushels of corn and x_1 is the amount of land used for corn. You also know that the production function for soybeans is $Y_2 = 100x_2^{1/2}$, where Y_2 is the amount of bushels of soybeans and x_2 is the amount of land used for soybeans. **(40 Points)**
- a. Please find the production possibility frontier (PPF) using soybeans as the dependent variable and corn as the independent variable?
 - b. What is the maximum amount of corn you can produce?
 - c. What is the maximum amount of soybeans you can produce?
 - d. How many bushels of soybeans can you produce if you produced 300 bushels of corn?
 - e. Please sketch the PPF with corn on the x-axis and soybeans on the y-axis.
 - f. What is the marginal rate of product transformation (MRPT) for the PPF?
 - g. What is the slope, i.e., MRPT, of the PPF when corn = 300, i.e., $Y_1 = 300$.
 - h. Please explain what part g means in an economic sense?