Ag Bus 313
Midterm 1
2/14/19
Section 2
Dr. Hurley

General Instructions: This exam is worth **200 points**. You must provide your own paper. You are allowed one 3x5 note card written on one side for the exam. This note card can have anything on it but if it is larger than 3x5 you will get a zero on the exam. You are allowed to use a calculator. You must show all your work when appropriate to get credit. This includes showing all applicable formulas you use. No cell phones, music players (ipods), or tablets are allowed to be in your possession during the exam. If you are caught with any of these devices, you will receive a zero on the exam. ***Any exam material left visible and unattended, or visible and on the ground will be thrown out by the professor when discovered.***

1. If your average physical product (APP) is equal to 5 using 10 inputs, what do you expect to happen to your current APP when your marginal physical product (MPP) is 5 when you move to 14 inputs? What is your new APP given this new MPP? Please show how you found your answer. **(15 points)**

2. Please explain the argument why marginal cost can equal both the change in total cost divided by the change in output and the change in total variable cost divided by the change in output when total cost and variable cost are not equal? **(5 points)**

3. Suppose your goal this year is to produce 6,250 bushels of organic corn. Suppose your production technology has the following relationship for producing bushels of corn Q = f(L,T) = LT where Q is the number of bushels of corn you produce, L is the number of labor hours you utilize, and T is the number of tractor hours you utilize. You know that the cost per hour of labor is $15 and the cost per hour of tractor time is $150. Assume that you are a cost minimizing producer.

   a) How much money will you need to ask from your banker to achieve your goal? (Note: when given your particular production function, the optimal amount of input formulas are $L = \sqrt{Q\frac{p_T}{p_L}}$ and $T = \sqrt{Q\frac{p_L}{p_T}}$, where p_L is the cost of labor and p_T is the cost of tractor time.) **(5 points)**

   b) Please sketch a graph of this solution. Include the isoquant and iso-cost line. **(10 points)**

4. Solve x_2 as a function of x_1: **(5 Points)**

   a) $4x_1^{2/5} x_2^{-2/5} + 7 = x_1^{-13/5} x_2^{3/5} + 7$

5. Solve the following for Y_2 as a function of Y_1: **(5 Points)**

   a) $\frac{Y_1^3}{125} + \frac{Y_2^6}{15625} = 64$

6. Using the three equations, get Y_2 as a function of Y_1 (Please do not represent your answer in decimals.): **(15 Points)**

   a) $Y_1 = 9x_1^{2/3}$
      $Y_2 = 36x_2^{2/3}$
      $x_1 + x_2 = 125$

7. Find the inverse of the following functions: **(5 Points)**

   a) $y = f(x) = 4x^2 + 64x + 256$

8. Solve for Y_1: **(10 Points)**

   a) $\frac{15Y_1^{1/3}}{\left(54000-275Y_1^{2/3}\right)^{1/2}} = \frac{6}{14}$

9. Using limits, find the general slope of the following: **(15 Points)**

   a) $y = f(x_1, x_2) = 27x_1 + 18x_1x_2^4 + 77x_2^3 + 2{,}718$ (For this problem, you need to apply the limit formula for **$\underline{x_1}$**.)

10. Please find the derivative of the following functions: **(6 Points)**
    a) $y = f(x) = 9x^5 + 15x^3 + 180x^{-1/4} + 45$
    b) $y= f(x) = (3x^2 – 27) / (x + 3)$ (Use the Quotient Rule)
    c) $y= f(x) = (7x^2 + 7x + 7)^{-1/7}$ (Use either Generalized Power Rule or the Chain Rule)

11. Find the first order and second order (this includes the cross partial derivatives) partial derivatives of the following functions with respect to **$\underline{x_2}$**: **(9 Points)**[1]

    a) $y = f(x_1,x_2) = 15x_1^2 + 18x_1^2x_2^3 + 60x_2^3 + 9x_1^{1/3}x_2^{2/3} + (27x_1x_2)^{2/3}$

12. Use the first order conditions to find the critical points of the function. Use the second order conditions to show whether the critical points are maximum, minimum, or saddle points (point of inflection). Are the critical points relative or absolute extrema? Please explain. **(20 Points)**

    a) $y = f(x) = –2(64 – x^2)^2$

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[1] You need to find: $\frac{\partial y}{\partial x_2}, \frac{\partial^2 y}{\partial x_2{}^2}$ and $\frac{\partial^2 y}{\partial x_2 \partial x_1}$

13. Maximize $f(x_1,x_2) = x_1^{1/4}x_2^{1/2}$ subject to the constraint $46{,}875 = g(x_1,x_2) = 25x_1 + 2x_2$. Solve this problem using the Lagrange Method or by changing it into an unconstrained maximization problem. **(20 Points)**

14. Answer the following questions based on the following production function:
    $y = f(x) = -6x^3 + 900x^2$. Please show how you found your answer. **(25 Points)**

    a) What is the marginal physical product (MPP) function for this production function?

    b) In terms of inputs, show where the MPP function is at a maximum.

    c) Demonstrate that MPP is at a maximum by using derivatives.

    d) What is the highest achievable MPP?

    e) Where does Stage I of the production function turn into Stage II of the production function?

15. Answer the following questions based on the following production function:
    $y = f(x_1,x_2) = -x_1^2 + 20x_1 - x_2^2 + 10x_2$. Please show how you found your answer. **(30 Points)**

    a) Find the isoquant for any given output level? (Note: make sure you find the one that makes economic sense.)

    b) If output is equal to 75 and input 1 is equal to 3, how much input 2 do you need?

    c) Calculate the marginal rate of technical substitution (MRTS) in terms of inputs by taking the derivative of the isoquant you found in part a. **(10 Points)**

    d) Given the information in part b and c, what is the MRTS?

    e) Please explain what part d means in an economic sense?