Ag Bus 313
Midterm 1
2/15/17
Dr. Hurley

General Instructions: This exam is worth **200 points**. You must provide your own paper. You are allowed one 3x5 note card written on one side for the exam. This note card can have anything on it but if it is larger than 3x5 you will get a zero on the exam. You are allowed to use a calculator. You must show all your work when appropriate to get credit. <u>This includes showing all applicable formulas you use</u>. No cell phones, music players (ipods), or PDA's are allowed to be in your possession during the exam. If you are caught with any of these devices, you will receive a zero on the exam.

1. If your average physical product (APP) is equal to 5 using 20 inputs, what do you expect to happen to your current APP when your marginal physical product (MPP) is 20 when you move to 25 inputs? What is your new APP given this new MPP? Please show how you found your answer. **(5 points)**

2. Please explain how you decide what level of outputs you produce given a fixed level of an input. Show this on a graph. **(5 points)**

3. Suppose your goal this year is to produce 40,000 bushels of organic corn. Suppose your production technology has the following relationship for producing bushels of corn Q = f(L,T) = LT where Q is the number of bushels of corn you produce, L is the number of labor hours you utilize, and T is the number of tractor hours you utilize. You know that the cost per hour of labor is $15 and the cost per hour of tractor time is $960. Assume that you are a cost minimizing producer. **(15 points)**

   a) How much money will you need to ask from your banker to achieve your goal? (Note: when given your particular production function, the optimal amount of input formulas are $L = \sqrt{Q\frac{p_T}{p_L}}$ and $T = \sqrt{Q\frac{p_L}{p_T}}$, where p_L is the cost of labor and p_T is the cost of tractor time.)

   b) Please sketch a graph of this solution. Include the isoquant and iso-cost line. **(10 Points)**

4. Solve x_2 as a function of x_1: **(5 Points)**

   $8x_1^{-4/3}\, x_2^{4/3} = 32x_1^{8/3}\, x_2^{-2/3}$

5. Solve the following for Y_2 as a function of Y_1: **(5 Points)**

   $$\frac{Y_1^6}{27} + \frac{Y_2^3}{216} = 60$$

6. Using the three equations, get Y_2 as a function of Y_1 (Please do not represent your answer in decimals.): **(10 Points)**

   $Y_1 = 25x_1^{2/3}$
   $Y_2 = 400x_2^{2/3}$
   $x_1 + x_2 = 8{,}000$

7. Find the inverse of the following functions: **(5 Points)**

   $y = f(x) = 36x^2 + 360x + 900$

8. Solve for Y_1: **(10 Points)**

   $$\frac{2Y_1^{1/2}}{(1{,}216 - 8Y_1^{3/2})^{1/3}} = \frac{5}{3}$$

9. Using limits, find the general slope of the following: **(10 Points)**

   $y = f(x_1, x_2) = 42x_1 + 90x_1x_2 + 22x_2^2 + 4{,}290$ (For this problem, you need to apply the limit formula for **$\underline{x_1}$** only.)

10. Please find the derivative of the following functions: **(10 Points)**

    a) $y = f(x) = 10x^{1/5} + 11x^{11} + 8x^{-1/2} - 21{,}214$

    b) $y = f(x) = (4x^2 - 8x - 60)/(5 - x)$

    c) $y = f(x) = (9x^3 + 27x + 2727)^{-1/3}$ (Use either Generalized Power Rule or the Chain Rule)

11. Find the first order and second order (this includes the cross partial derivatives) partial derivatives of the following functions with respect to **$\underline{x_2}$**: **(10 Points)**[1]

    $y = f(x_1,x_2) = 16x_1^{1/4} + 12x_2^3 + 48x_1^{1/4}x_2^{3/4} + 100(x_1x_2)^{1/2}$

12. Use the first order conditions to find the critical points of the function. Use the second order conditions to show whether the critical points are maximum, minimum, or saddle points (point of inflection). **(20 Points)**

    $y = f(x) = -(25-x^2)^2$

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[1] The cross derivatives for a function $f(x_1,x_2)$ are $\frac{\partial^2 f(x_1,x_2)}{\partial x_1 \partial x_2}$ and $\frac{\partial^2 f(x_1,x_2)}{\partial x_2 \partial x_1}$

13. Maximize $f(x_1,x_2) = x_1^{1/2}x_2^{1/4}$ subject to the constraint $5{,}400 = g(x_1,x_2) = 150x_1 + 50x_2$. Solve this problem using the Lagrange Method **or** by changing it into an unconstrained maximization problem. **(20 Points)**

14. Answer the following questions based on the following production function: $y = f(x) = -15x^3+1800x^2$. Please show how you found your answer. **(15 Points)**

    a. What is the average physical product (APP) function for this production function?

    b. In terms of inputs, show where the APP function is at a maximum. Demonstrate that APP is at a maximum by using derivatives. What is the highest achievable APP?

    c. Where does Stage II of the production function turn into Stage III of the production function? (Give me an actual number)

15. Answer the following questions based on the following production function: $y = f(x_1,x_2) = -5x_1^2 + 80x_1 - 12x_2^2 + 96x_2$. Please show how you found your answer. **(15 Points)**

    a. What is the marginal physical product (MPP_{x1}) function for this production function with respect to input 1? What is the marginal physical product (MPP_{x2}) function for this production function with respect to input 2?

    b. What is the extrema point to this production function? What is the maximum you can achieve for output? **(10 points)**

16. Answer the following questions based on the following production function: $y = f(x_1,x_2) = 20x_1^{2/3}x_2^{1/3}$. Please show how you found your answer. **(25 Points)**

    a. Find the isoquant for any given output level.

    b. If output is equal to 1,080 and input 1 is equal to 27, how much input 2 do you need?

    c. Calculate the marginal rate of technical substitution (MRTS) in terms of inputs by taking the derivative of the isoquant you found in part a. **(10 points)**

    d. What are the returns to scale for this production function?

17. Suppose you have 9 acres of land to allocate to corn and soybeans. The production function for corn is $Y_1 = 27x_1^{1/3}$, where Y_1 is the amount of bushels of corn and x_1 is the amount of land used for corn. You also know that the production function for soybeans is $Y_2 = 216x_2^{1/3}$, where Y_2 is the amount of bushels of soybeans and x_2 is the amount of land used for soybeans. **(15 Points)**

    a. Please find the production possibility frontier (PPF) using soybeans as the dependent variable and corn as the independent variable? **(15 Points)**