Consumer Choice, Utility, and Revealed Preference

Agenda

- Consumer Utility
- Consumer Choice
- Revealed Preference

Consumer Theory

- There are two important pillars that consumer theory rests upon:
  - The utility function
  - The budget constraint

Utility Function

- A utility function is a function/process that maps a bundle of goods into satisfaction where the satisfaction can be ranked for each bundle of goods.
  - In essence, it allows us to rank the desirability of consuming different bundles of goods.
- A bundle of goods is a particular set of goods you choose to consume.
  - This is also referred to as a consumption bundle.

Mathematically Representing Utility

- In Economics, we tend to define the utility function as the following:
  - $U = u(x_1, x_2, ..., x_n)$ where
    - U is the level of satisfaction you receive from consuming the bundle of goods consisting of $x_1, x_2, ..., x_n$
    - u( ) is the function that maps the bundle of goods into satisfaction. (Think of this as the mathematical representation of your brain.)
    - x_i for i = 1, 2, ..., n is the quantity of a particular good consumed from a bundle of goods, e.g., x_1 may be 5 pancakes.

Example of Representing Utility in Mathematical Terms

- Suppose you have a utility function that is based on three goods pizza, soda, and buffalo wings. We can represent your utility function as $U = u(x_1, x_2, x_3)$.
  - x_1 = amount of pizza consumed
  - x_2 = amount of soda consumed
  - $x_n = x_3$ = amount of buffalo wings consumed

Example of Representing Utility in Mathematical Terms

- Suppose you have a choice of consuming one of two different bundles of goods.
  - Bundle 1 has a large pepperoni pizza, a two liter bottle of coke, and 20 spicy buffalo wings.
  - Bundle 2 has a large pepperoni pizza, a two liter bottle of coke, and 40 spicy buffalo wings.

Example of Representing Utility Cont.

- For bundle 1
  - x_1 = a large pepperoni pizza
  - x_2 = a two liter bottle of coke
  - $x_n = x_3$ = 20 spicy buffalo wings
- For bundle 2
  - x_1 = a large pepperoni pizza
  - x_2 = a two liter bottle of coke
  - $x_n = x_3$ = 40 spicy buffalo wings

Example of Representing Utility Cont.

- We can further write the following:
  - U^1 = u(a large pepperoni pizza, a two liter bottle of coke, 20 spicy buffalo wings)
  - U^2 = u(a large pepperoni pizza, a two liter bottle of coke, 40 spicy buffalo wings)
    - Remember u( ) is just a function that transforms bundles of goods into satisfaction. (Think of it as your brain.)
  - What might be inclined to say that $U^2 > U^1$.
    - Why?

Some Notes on Utility

- It was once thought that utility could be broken down into units called utils.
- Total utility is defined as the utility you receive from consuming a particular bundle of goods.
- For analytical tractability, total utility can be given a number value.
  - In our previous example we could say that $U^1 = 100$ and $U^2 = 200$.

Some Notes on Utility Cont.

- There are two views of utility:
  - Cardinal Utility
    - Cardinal utility is the belief that utility can be measured and compared on a unit by unit basis.
      - E.g., A utility measure of 200 is twice as big as a utility measure of 100.
  - Ordinal Utility
    - Ordinal utility is where you rank bundles of goods, but cannot say how much greater one bundle is to another.
      - Ranking is the only thing that matters when dealing with ordinal utility.

Utility from Consuming One Good

- When examining utility, we sometimes would like to examine how one good affects our utility.
- This can be looked at by assuming that we have only one good in our utility function, i.e., $U=u(x)$.
- A more realistic outlook is that we examine only one good in our utility function, holding all the other goods constant.

Utility from Consuming One Good Cont.

- Mathematically, we can represent holding things constant in the following two manners for one good:
  - $U = u(x_1; x_2, x_3, ..., x_n)$ or
  - $U = u(x_1 \mid x_2 , x_3, ..., x_n)$
    - Where it is understood using this notation that goods x_2 through x_n are held at some constant level.

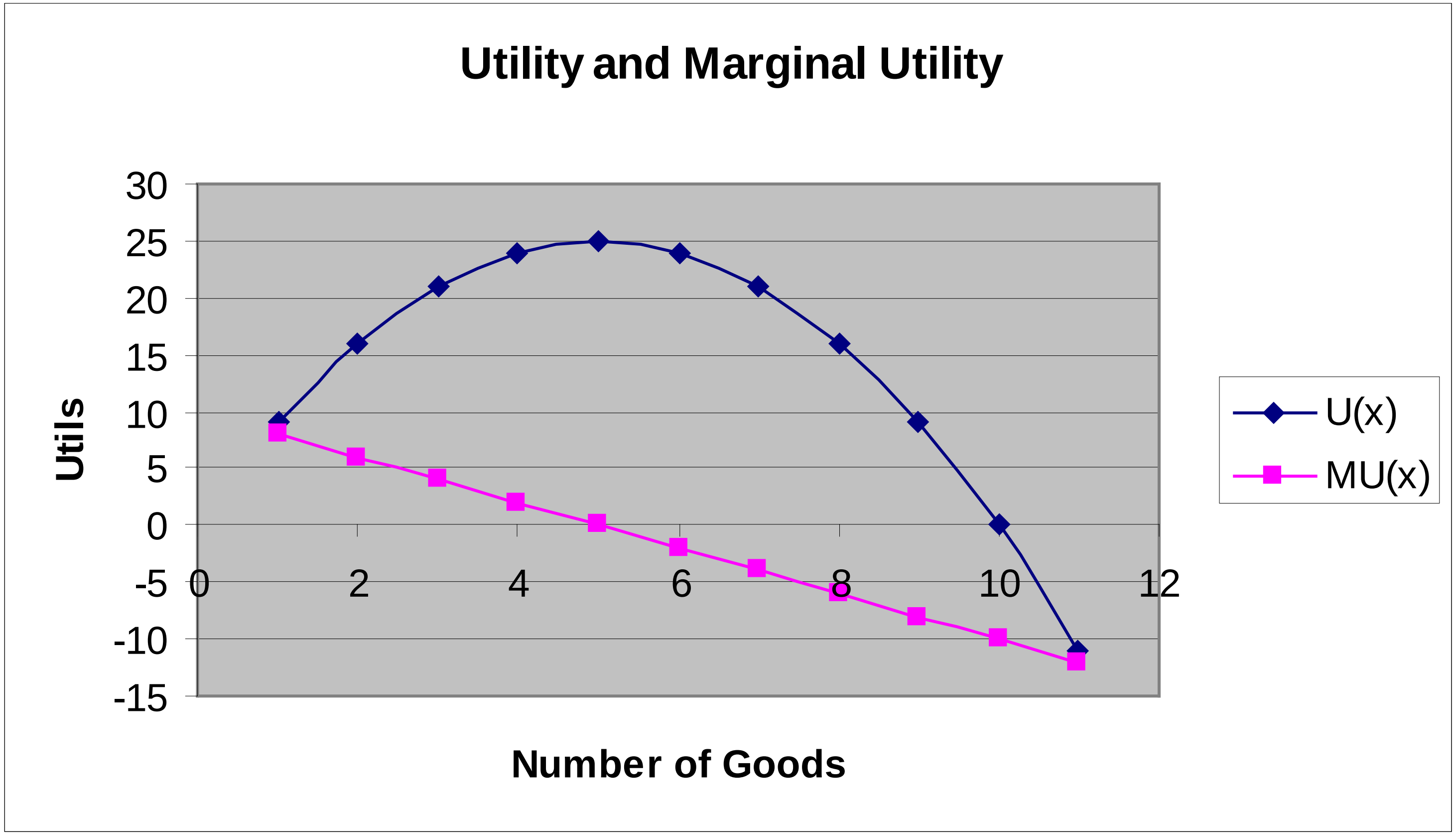
Marginal Utility (MU)

- Marginal utility is defined as the change in total utility divided by a change in the consumption of a particular good.
- In mathematical terms, marginal utility of good i is defined as du(x)/dx, which is just the first derivative of the utility function with respect to good x.

Calculating Marginal Utility

- Suppose Dr. Hurley only like to consume chocolate.
- Also, assume that Dr. Hurley's total utility function can be represented as $U(x) = 10x - x^2$
- This implies that $MU(x)=du/dx= 10 - 2x$
  - Assuming that chocolates were free, how many chocolates would Dr. Hurley consume?

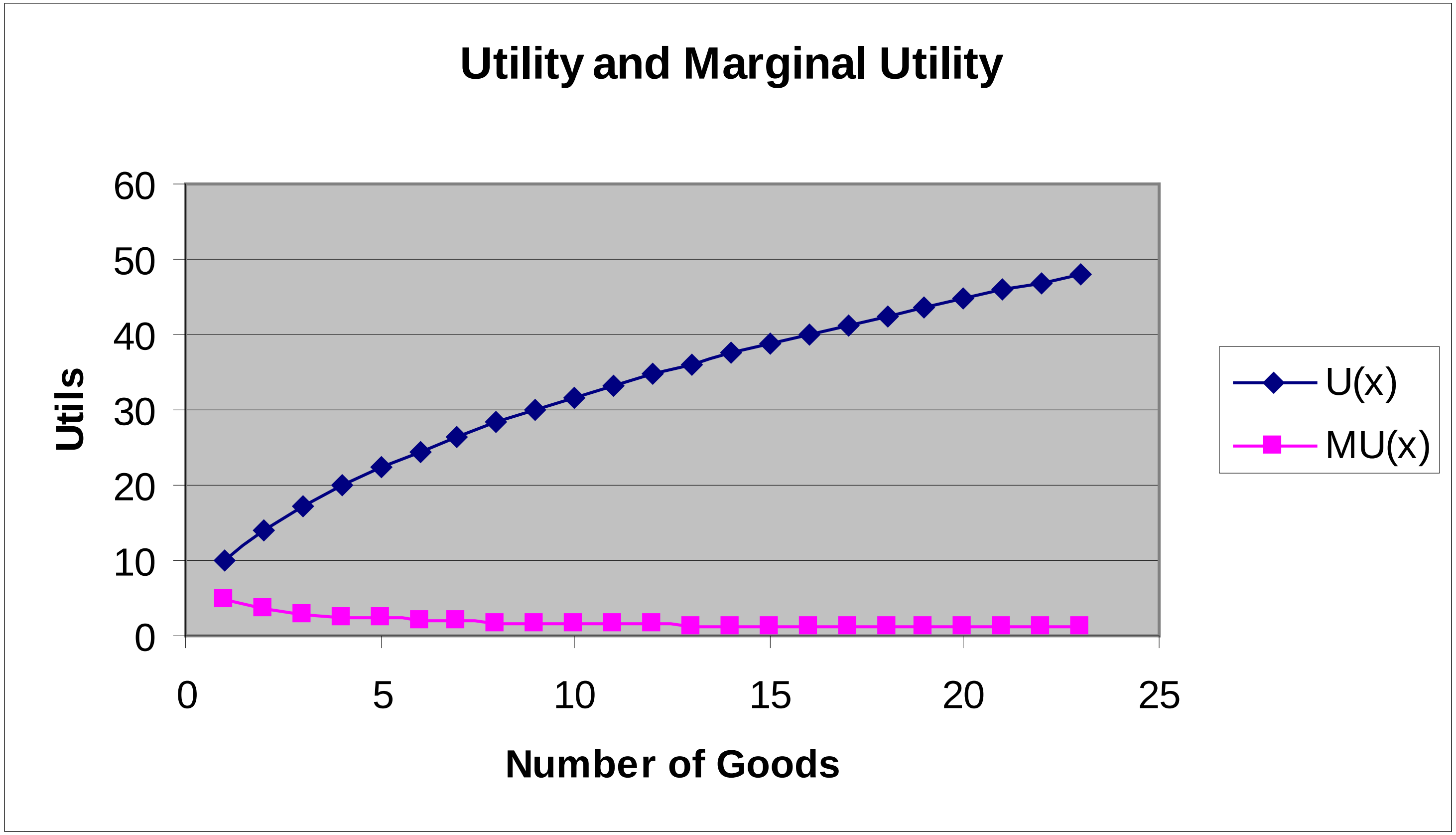
Total Utility and marginal Utility of $U(x)=10x-x^2$



Deriving Total and Marginal Utility from a Function

- Suppose you could represent your utility function for drinking Mountain Dew as the following: $U = u(x)$
  - Where $u(x) = 10x^{½}$
- What is Total Utility and Marginal Utility?
  - Graph both.

Total Utility and marginal Utility of $U(x)=10x^{½}$



Lessons Learned from Example

- As long as marginal utility is positive, total utility will increase.
- Total and marginal utility can be negative.
- When marginal utility is zero, total utility is maximized.
- This graph demonstrated the Law of Diminishing Marginal Utility.

Law of Diminishing Marginal Utility

- This law states that as you consume more units or a particular good during a set time, at some point your marginal utility will decrease as your consumption increases.
  - Note: Beware of laws in economics, they are not like physical laws.
  - This is equivalent to saying that the derivative of MU is negative.

Utility from Consuming Two Goods

- When examining utility, we sometimes would like to examine how two good interrelate in our utility function.
- This can be looked at by assuming that we have only one good in our utility function, i.e., $U=u(x_1, x_2)$.
- A more realistic outlook is that we examine two goods in our utility function, holding all the other goods constant.

Utility from Consuming Two Goods Cont.

- Mathematically, we can represent holding things constant in the following two manners for two goods:
  - $U = u(x_1, x_2; x_3, ..., x_n)$ or
  - $U = u(x_1, x_2 \mid x_3, ..., x_n)$
    - Where it is understood using this notation that goods x_3 through x_n are held at some constant level.

Isoutility

- To this point we have examined utility based on the consumption of one good.
  - Realistically, our consumption bundles have more than one item in them.
- When more than one good exist in our consumption bundle and utility function, it becomes important for us to examine the idea of isoutility.

Isoutility

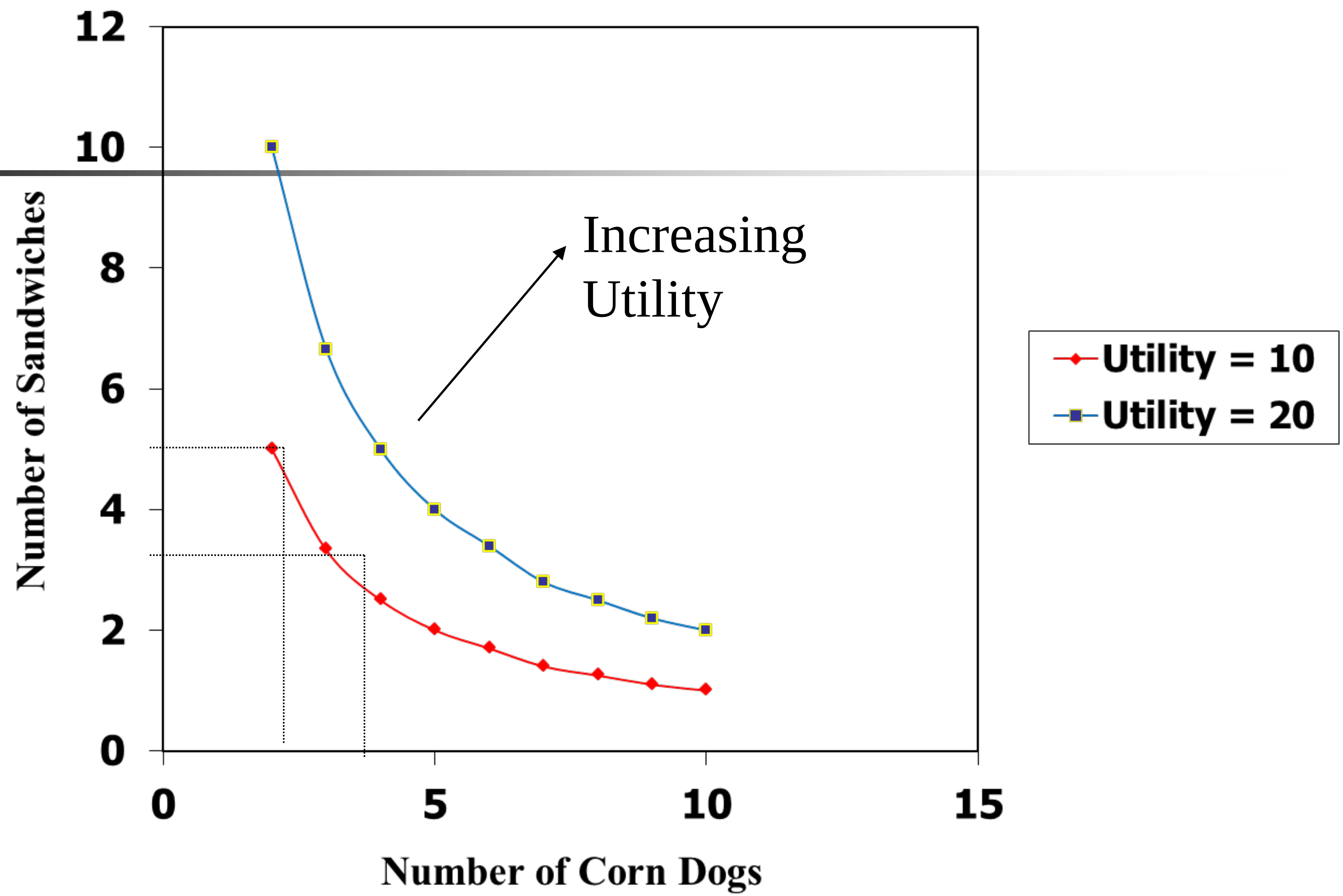
- Isoutility is a concept where differing bundles of goods provide the same level of utility.
- From the idea of isoutility comes the graphical representation--an indifference curve.
  - An indifference curve is the graphical representation of differing bundles of goods giving the same level of utility.

Demonstrating an Indifference Curve

- Suppose we have two goods in our utility function—corn dogs (C) and sandwiches (S).
- To obtain an indifference curve, we want to hold our utility constant and look at the different bundles of consumption of each good that provide the same utility.
- Suppose we know that we have the following utility function $U = u(C, S) = C*S$

Demonstrating an Indifference Curve Cont.

- To derive the indifference curve, we want to keep U constant.
  - In this case we will assume U = 10
    - What bundles of goods will give us this level of satisfaction?
    - What if U = 20?



12
10
8
6
4
2
0
Number of Sandwiches
0
5
10
15
Number of Corn Dogs
Increasing
Utility
Utility = 10
Utility = 20

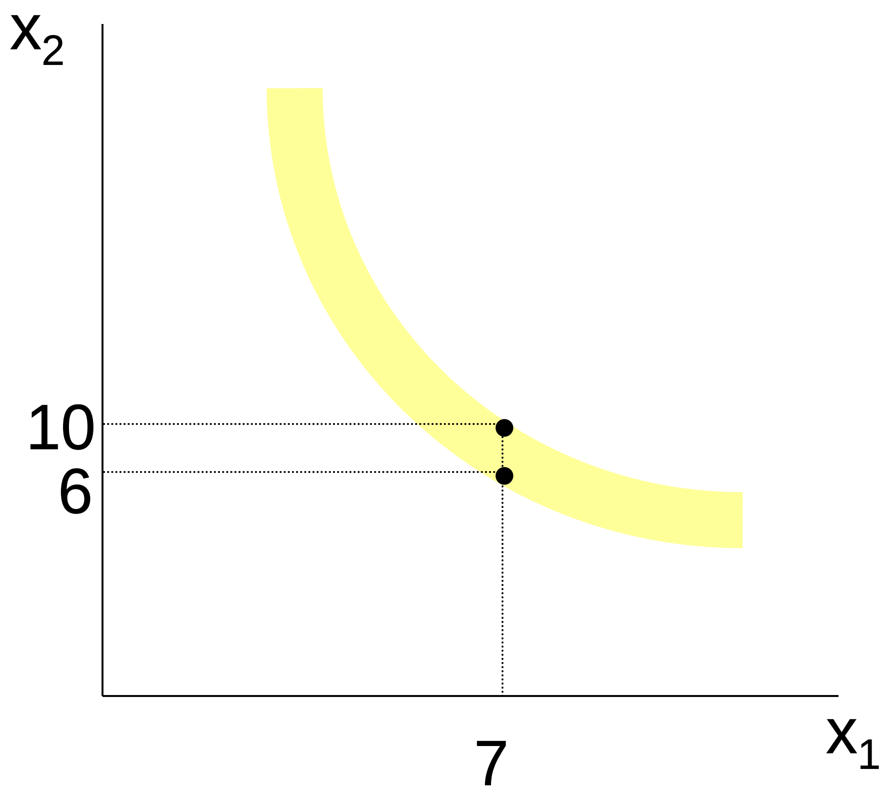
Observations

- The two utility curves do not cross each other.
  - Why?
- Utility is increasing as you move away from the x and y-axis.
- As you consume more of one of the goods, the consumption of the other good has relatively more value to you.
- Any other observations?

Notes on Indifference Curves

- Indifference curves usually are not considered thick.
  - Why.
- Indifference curves tend to not be upward sloping.
  - Why?

Example of a Thick Indifference Curve



Marginal Rate of Substitution (MRS)

- When examining indifference curves, it is important to look at the tradeoff between consuming one good versus the other.
  - This is called examining the marginal rate of substitution.
  - The marginal rate of substitution can be defined as the rate at which the consumer is willing to trade one good for another.
  - Why is this important to examine?

Mathematical Representation of MRS

- MRS of good x_i for good x_j
  - $=dx_j / dx_i$

Example of Calculating MRS

- From our previous example, we had the following utility function:
  - U = u(C,S) = C*S
  - Setting U = 10, what is the MRS of corndogs for sandwiches at 2 sandwiches and 5 corndogs, i.e., dS /dC?

Implication of Indifference Curve

- Due to indifference curves, we have the following relationship for differing bundles of goods:
  - $(dx_j \ / \ dx_i) = -(MU_{xi} \ / MU_{xj})$
    - Which implies $(dx_j * MU_{xj}) = -(MU_{xi} * dx_i)$
    - Which implies $(dx_j * MU_{xj}) + (MU_{xi} * dx_i) = 0$
  - This implies that the loss in utility from consuming less of good x_j is just matched by the gain in utility from consuming more of x_i.

Budget Constraint

- This constraint defines the feasible set of consumption bundles you are able to consume.
- It is dependent on three things:
  - The prices of the goods in your consumption bundle.
  - The quantity of each good in your consumption bundle.
  - Your income.

Budget Constraint Cont.

- The budget constraint for two goods is defined as the following:
- $m = p_1*x_1 + p_2 * x_2$
  - m = income
  - p_1 = the price of x_1
  - p_2 = the price of x_2
  - x_1 = the quantity consumed of good 1
  - x_2 = the quantity consumed of good 2

Budget Constraint Cont.

- Assume that we want to graph are budget constraint and choose x_2 as the good we want to put on the y-axis.
- $m = p_1*x_1 + p_2 * x_2$ can be rewritten as:
- $x_2 = (m/p_2) – (p_1/p_2) * x_1$
  - With this equation we can draw the budget constraint on a graph.
  - Note that the slope of the budget constraint is equal to the negative of the price of good 1 divided by the price of good 2.

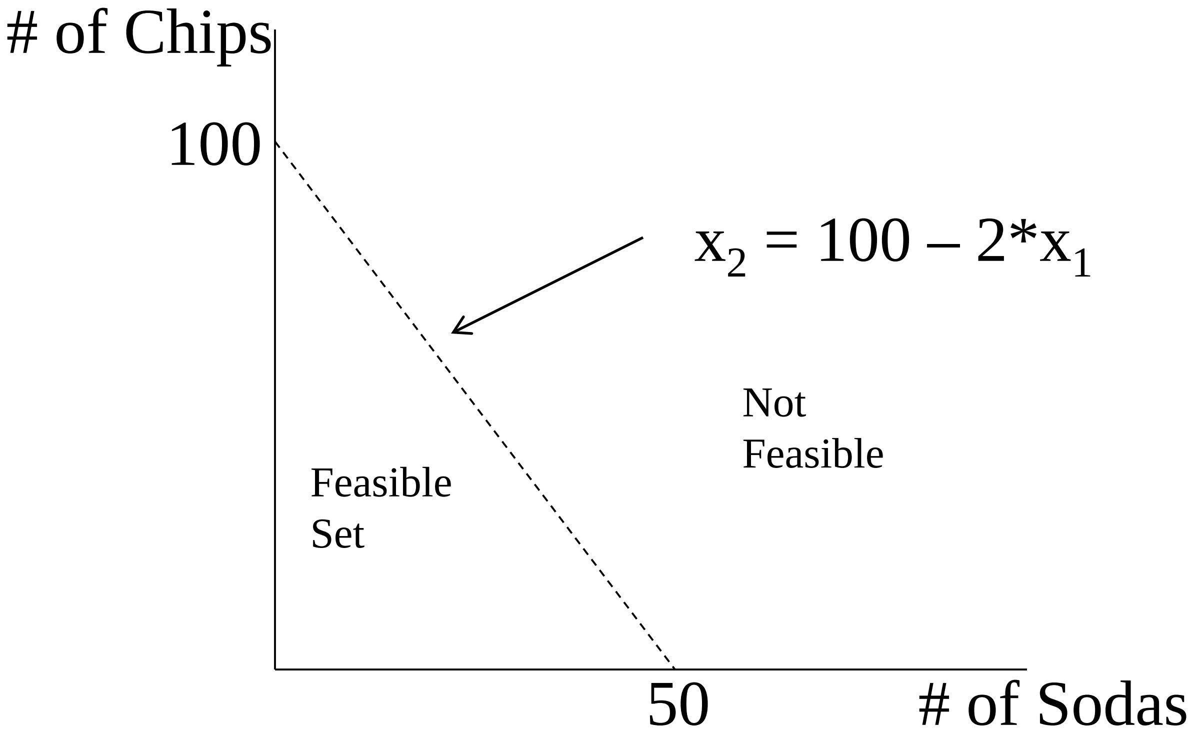
Budget Constraint Example

- Suppose you have $100 to spend on chips and soda. You know that the price of chips is $1 and the price of soda is $2. Draw your budget curve assuming that chips will be on the y-axis.

Budget Constraint Example

- Budget equation:
  - $\$100 = \$2 *x_1 + \$1 *x_2$
  - $\Rightarrow x_2 = 100 - 2*x_1$

Budget Constraint Cont.



Consumer Equilibrium

- Consumer equilibrium is comprised of two concepts:
  - The utility function
  - The budget constraint
- Consumer equilibrium can be defined as a consumption bundle that is feasible given a particular budget constraint and maximizes total utility.

Consumer Equilibrium Cont.

- If there was no budget constraint, a person would consume each good to the point where marginal utility of consumption for each good is zero.
  - Why?
- Given a budget constraint, the consumer maximizes total utility by consuming a bundle that is feasible.
  - A feasible bundle is one that lies either on or inside the budget constraint.

Consumer Equilibrium Cont.

- In graphical terms, consumer equilibrium is defined as the point where the highest utility function touches the budget constraint.

General Utility Maximization Model

$$\max_{x_1, x_2, \ldots, x_n} \; u(x_1, x_2, \ldots, x_n)$$

subject to:

$$m \geq \sum_{i=1}^{n} p_i x_i$$

Utility Maximization with One Good

$$\underset{x}{Max}\, u(x)$$

$$\text{subject to :}$$

$$m \geq px$$

$$\Gamma(x,\lambda) = u(x) + \lambda(m - px)$$

$$\frac{\partial \Gamma}{\partial x} = \frac{\partial u}{\partial x} - \lambda p = 0$$

$$\Rightarrow \lambda = \frac{\frac{\partial u}{\partial x}}{p} = \frac{MU}{p}$$

$$\frac{\partial \Gamma}{\partial \lambda} = m - px \geq 0$$

Utility Maximization with One Good Cont.

- When your maximizing utility with respect to one good, there are two scenarios that occur:
  - You have more money than you need to maximize your utility, so you do not spend all of your money.
    - This implies $\lambda = 0$ and $m > px$.
  - You run out of money before you reach the highest amount of utility you can achieve.
    - This implies $\lambda > 0$ and $m = px$.
  - To solve this problem, you need to check which scenario provides the highest utility while satisfying the constraint.

Consumer Equilibrium Example for One Good

- Suppose we have the following utility function:
  - $U = u(x) = x^{1/2}$
- Suppose the price of good x is $5 and the income available is $125.
- What is the optimal utility and how much money do you have left?

Consumer Equilibrium Example for One Good Cont.

- Now, Suppose we have the following utility function:
  - $U = u(x) = 10x - x^2$
- Suppose the price of good x is $5 and the income available is $125.
- What is the optimal utility and how much money do you have left?

Utility Maximization for Two Goods

- Suppose we have the following utility function:
  - $U = u(x_1,x_2)$
    - Where x_1 is equal to the number of good 1
    - Where x_2 is equal to the number of good 2
- Suppose we have the following budget constraint:
  - $M = p_1*x_1 + p_2*x_2$
    - Where p_1 is equal to the price of good 1
    - Where p_1 is equal to the price of good 2

Utility Maximization with Two Goods Cont.

$$\underset{x_1, x_2}{Max}\, u(x_1, x_2)$$

$$\text{subject to} : m = p_1 x_1 + p_2 x_2$$

Utility Maximization with Two Goods Cont.

$$\Gamma(x_1, x_2, \lambda) = u(x_1, x_2) + \lambda(m - p_1 x_1 - p_2 x_2)$$

$$\frac{\partial \Gamma}{\partial x_1} = \frac{\partial u}{\partial x_1} - \lambda p_1 = 0$$

$$\Rightarrow MU_{x_1} = \lambda p_1$$

$$\Rightarrow \lambda = \frac{MU_{x_1}}{p_1}$$

$$\frac{\partial \Gamma}{\partial x_2} = \frac{\partial u}{\partial x_2} - \lambda p_2 = 0$$

$$\Rightarrow MU_{x_2} = \lambda p_2$$

$$\Rightarrow \lambda = \frac{MU_{x_2}}{p_2}$$

Utility Maximization with Two Goods Cont.

- $(p_j/p_i) = (MU_j/MU_i)$ can be rewritten as:
  - $(MU_j / p_j) = (MU_i / p_i)$
  - This says that you are normalizing the change in utility by the price of the good and then equating it to the normalized marginal utility of the other good.
- Another way to look at this is to say that the marginal utility derived from the last dollar spent for each good is equal.
- What happens if one side is greater than the other?

Utility Maximization with Two Goods Cont.

- Intuitively what we have done in the graph is equate the tradeoff from prices to the tradeoff in utility.
  - I.e., $(p_j/p_i) = (MU_j/MU_i)$
    - Where p_j is the price of good j and p_i is the price of good i
    - Where MU_j is the marginal utility of consuming good j and MU_i is the marginal utility of consuming good i

Consumer Equilibrium Example for Two Goods

- Suppose we have the following utility function:
  - $U = u(x_1, x_2) = x_1 * x_2$
    - Where x_1 is equal to the number of hotdogs consumed
    - Where x_2 is equal to the number of sodas consumed
- Suppose we have the following budget constraint:
  - $M = p_1 * x_1 + p_2 * x_2$
    - Where p_1 is equal to the price of hotdogs consumed
    - Where p_2 is equal to the price of sodas consumed

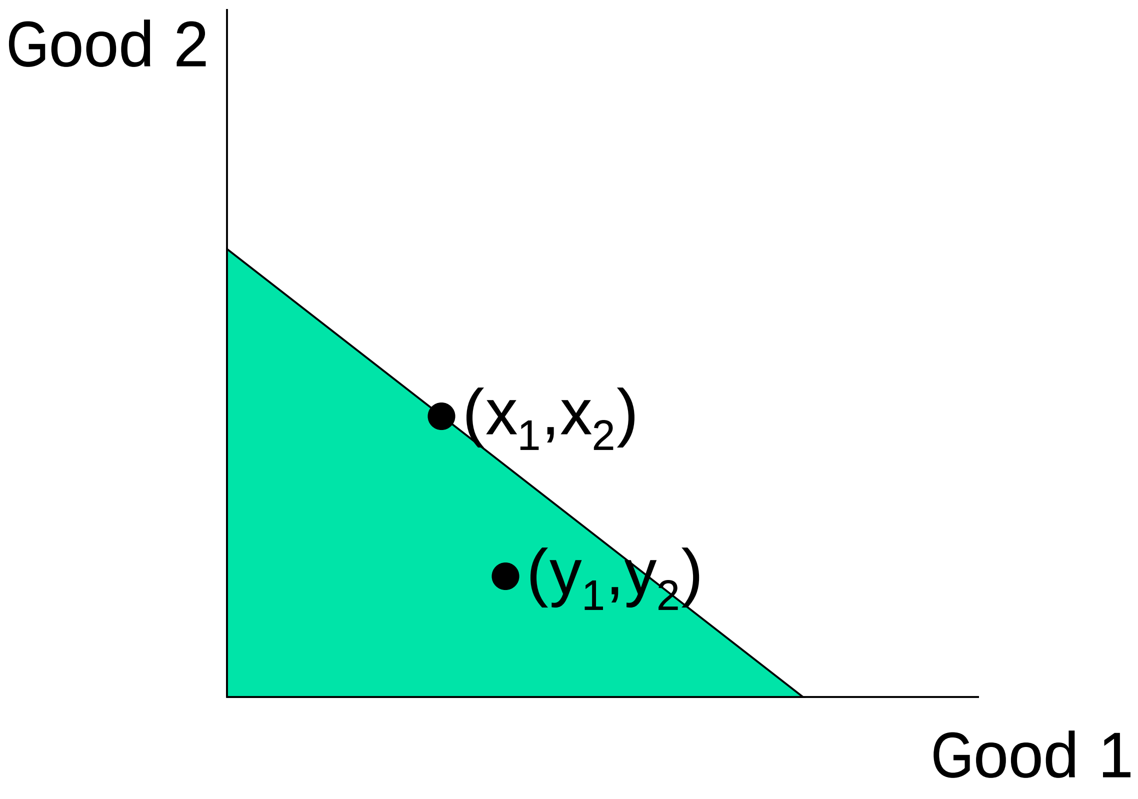
Consumer Equilibrium Example Cont.

- Now consider that you have a price of hotdogs (x_1) equal to $2 and a price of soda ($x_2$) is a $1.
- Also suppose that our income is $16.
- Solution will be done in class.

Revealed Preference

- Revealed Preference is about making observations of people's consumption values to understand their preferences.
- Suppose there are two bundles of goods (x_1,x_2) and (y_1,y_2).
- An assumption of revealed preference is that the consumer always chooses the optimal bundle given her budget constraint.
- If prices are p_1 and p_2, then we can represent these two bundles on a graph.

Revealed Preference Cont.



Revealed Preference Cont.

- Define $m = p_1x_1+p_2x_2$
- We know from the above graph that $p_1y_1+p_2y_2 \leq m$.
  - Why?
- We also know that $p_1x_1+p_2x_2 \geq p_1y_1+p_2y_2$.
  - Why?

The Principle of Revealed Preference

- If (x_1,x_2) is the chosen bundle of goods at prices (p_1,p_2), and if (y_1,y_2) is affordable, i.e., $p_1x_1+p_2x_2 \geq p_1y_1+p_2y_2$, then the bundle (x_1,x_2) is preferred to (y_1,y_2), i.e., $(x_1,x_2) > (y_1,y_2)$.
  - In this case, we say (x_1,x_2) is directly revealed preferred to (y_1,y_2).
- Note: If the consumer is choosing the optimal bundle, then revealed preference will imply preference.

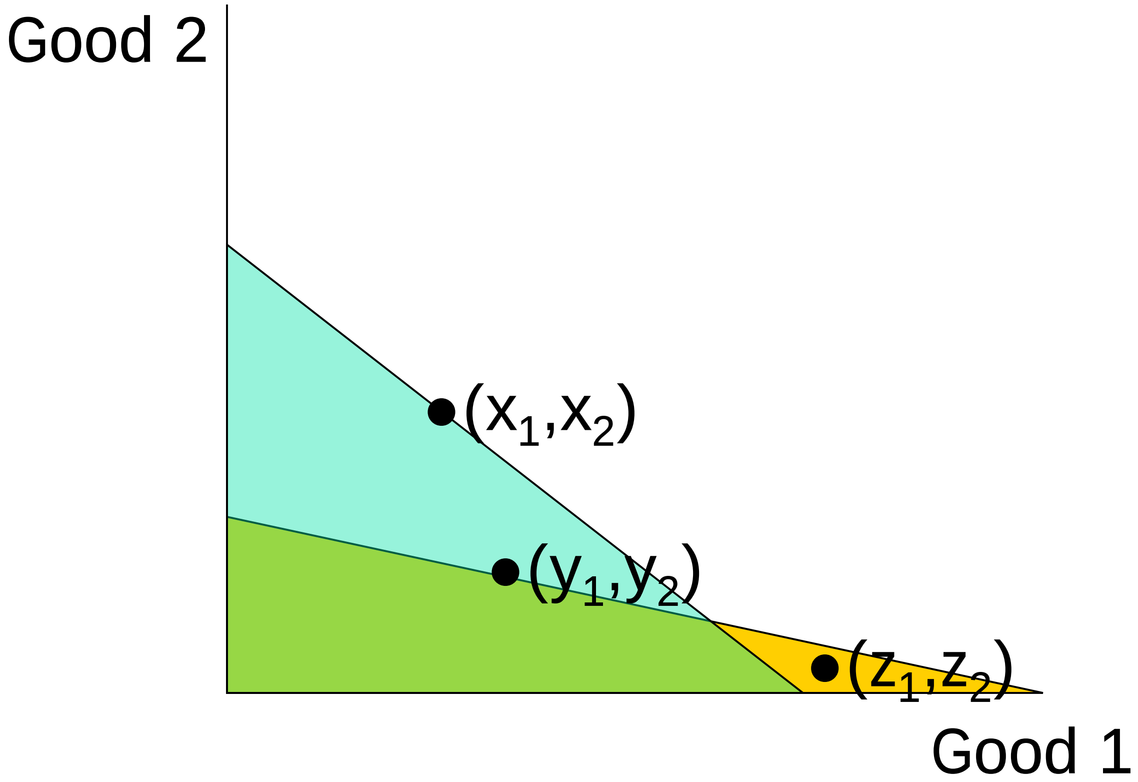
Indirectly Revealed Prefer

- Suppose that there is a set of prices (q_1,q_2) where (y_1,y_2) is directly revealed prefer to (z_1,z_2), i.e., the consumer chooses (y_1,y_2) while $q_1y_1+q_2y_2 \geq q_1z_1+q_2z_2$.
- Also assume that at price (p_1,p_2), (x_1,x_2) is directly revealed preferred to (y_1,y_2).

Indirectly Revealed Prefer

- Then we know that (x_1,x_2) is indirectly revealed preferred to (z_1,z_2), even if (z_1,z_2) was not affordable at (p_1,p_2).
- The graph on the next slide gives a visual representation of this.

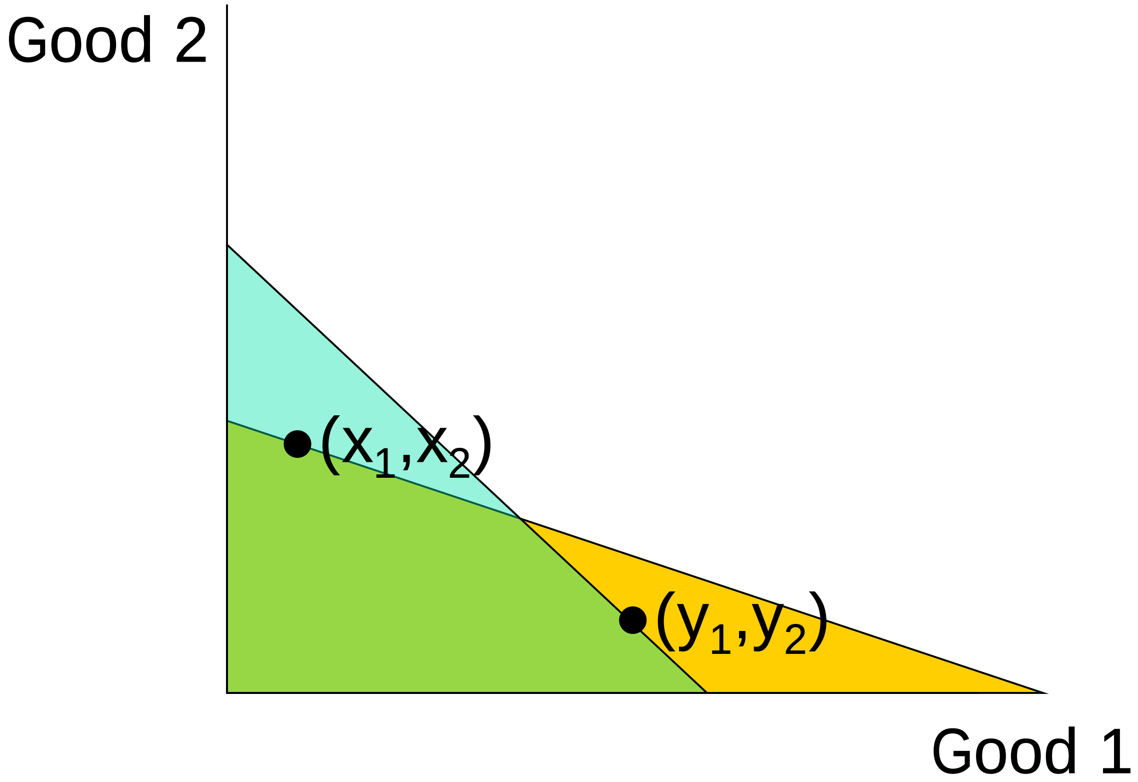
Indirectly Revealed Prefer Cont.



Weak Axiom of Revealed Preference (WARP)

- If the bundle (x_1,x_2) is directly revealed preferred to bundle (y_1,y_2) and the two bundles are not the same, then it is not possible that (y_1,y_2) is directly revealed preferred to (x_1,x_2) when (x_1,x_2) is affordable.

Graphical Example of WARP Being Violated



Example of WARP Being Violated

Observation	p_1	p_2	x_1	x_2
1	2	6	3	5
2	3	3	4	4

Example of WARP Being Violated Cont.

		Observations (x_1,x_2)	
		1 (3,5)	2 (5,4)
Prices (p_1,p_2)	1 (2,6)	36	34*
	2 (3,3)	24*	27

Strong Axiom of Revealed Preference

- If the bundle (x_1,x_2) is directly or indirectly revealed preferred to bundle (y_1,y_2) and the two bundles are not the same, then it is not possible that (y_1,y_2) is directly or indirectly revealed preferred to (x_1,x_2).

Example of SARP Being Violated

Observation	p_1	p_2	x_1	x_2
1	1	1	9	4
2	2	1	3	7
3	1	2	1	10

Example of SARP Being Violated Cont.

		Observations (x_1,x_2)		
		(9,4)	(3,7)	(1,10)
Prices (p_1,p_2)	(1,1)	13	10	11
	(2,1)	23	13	12
	(1,2)	17	17	21