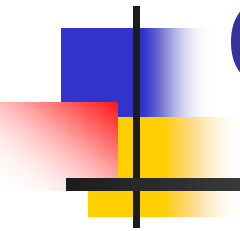
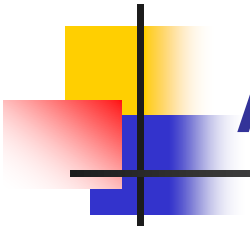
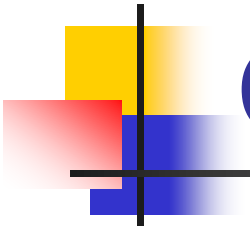


Game Theory



Agenda

- Game Theory
- Matrix Form of a Game
- Dominant Strategy and Dominated Strategy
- Nash Equilibrium
- Game Trees
- Subgame Perfection



Game Theory

- Game theory is the study of a set of tools that can be used to analyze decision-making by a set of players who interact with each other through a set of strategies.

Matrix Form of the Game

- The matrix form of a game represents the strategies and payoff of those strategies in a matrix.
  - The key components of this is the players, the strategies, and the payoffs of the strategies.
- The matrix form is a useful tool when the players of the game must move simultaneously.

Matrix Form of the Game Cont.

		Player 2	
		Strategy A	Strategy B
Player 1	Strategy 1	(Player 1 payoff, Player 2 payoff)	(Player 1 payoff, Player 2 payoff)
	Strategy 2	(Player 1 payoff, Player 2 payoff)	(Player 1 payoff, Player 2 payoff)

Matrix Form Example

- Suppose there were two producers of hogs, Farmer A and Farmer B.
- Assume that there are two strategies for farmer A: be large or be small.
- Assume that there are two strategies for farmer B: decrease size or increase size.
- If farmer A chooses small and Farmer B chooses decrease, they will evenly split 100.

Matrix Form Example Cont.

- If farmer A chooses large and Farmer B chooses increase, they will evenly split 80.
- If farmer A chooses large and Farmer B chooses decrease, then farmer A gets 60 and farmer B gets 20.
- If farmer A chooses small and Farmer B chooses increase, then farmer A gets 30 and farmer B gets 70.

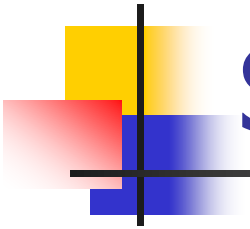
Matrix Form Example Cont.

- Players: Farmer A, Farmer B
- Strategies: Large, Small, Increase, Decrease
- Payoffs:
  - Large, increase implies (40, 40)
  - Large, decrease implies (60, 20)
  - Small, decrease implies (50, 50)
  - Small, increase implies (30, 70)

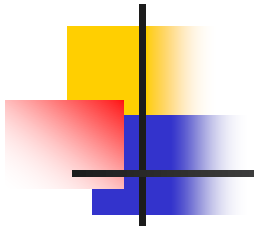
Matrix Form Example

		Farmer B	
		Decrease	Increase
Farmer A	Small	(50, 50)	(30, 70)
	Large	(60, 20)	(40, 40)

Dominant and Dominated Strategies



- Dominant Strategy
  - Given a set of strategies, a dominant strategy is one that is better than all other strategies in that set.
- Dominated Strategy
  - Given a set of strategies, a dominated strategy is one that is worse than some other strategy in that set.



- A strategy “dominates” another strategy when every payoff in the “dominating” strategy is larger than its corresponding payoff in the strategy that is being dominated.

Example to Help with Terminology

		Farmer B	
		Decrease	Increase
Farmer A	Small	(P_{SDA}, P_{SDB})	(P_{SIA}, P_{SIB})
	Large	(P_{LDA}, P_{LDB})	(P_{LIA}, P_{LIB})

Example to Help with Terminology Cont.

- Define:
  - P_{SDA} as the payoff to player A if she plays strategy small and player B plays strategy decrease.
  - P_{SDB} as the payoff to player B if he plays strategy decrease and player A plays strategy small.
  - P_{SIA} as the payoff to player A if she plays strategy small and player B plays strategy increase.
  - P_{SIB} as the payoff to player B if he plays strategy increase and player A plays strategy small.

Example to Help with Terminology Cont.

- P_{LDA} as the payoff to player A if she plays strategy large and player B plays strategy decrease.
- P_{LDB} as the payoff to player B if he plays strategy decrease and player A plays strategy large.
- P_{LIA} as the payoff to player A if she plays strategy large and player B plays strategy increase.
- L_{SIB} as the payoff to player B if he plays strategy increase and player A plays strategy large.

Example to Help with Terminology Cont.

- For Player A, the small strategy dominates the large strategy if $P_{SDA}>P_{LDA}$ and $P_{SIA}>P_{LIA}$.
  - Hence, large is said to be a dominated strategy.
  - Is small a dominant strategy?
  - What conditions would need to hold for decrease to be a dominated strategy?

Nash Equilibrium

- A Nash Equilibrium is said to occur when given the strategies of the other players are held constant, there is no incentive for a player to change his strategy to get a higher payoff.
  - In essence, a Nash Equilibrium occurs when all players in the game do not want to change their strategies given the other players' strategies.

Nash Equilibrium Cont.

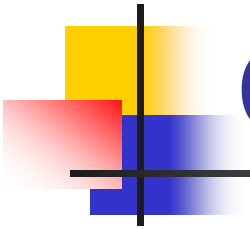
- A Nash Equilibrium will be in a Dominant Strategy.
- A Nash Equilibrium will never be in a dominated strategy.

Matrix Form Example with Nash Equilibrium

		Farmer B: Decrease	Farmer B: Increase
Farmer A	Small	(50, 50)	(30, 70)
	Large	(60, 20)	(40, 40) Nash Equilibrium

Prisoners' Dilemma

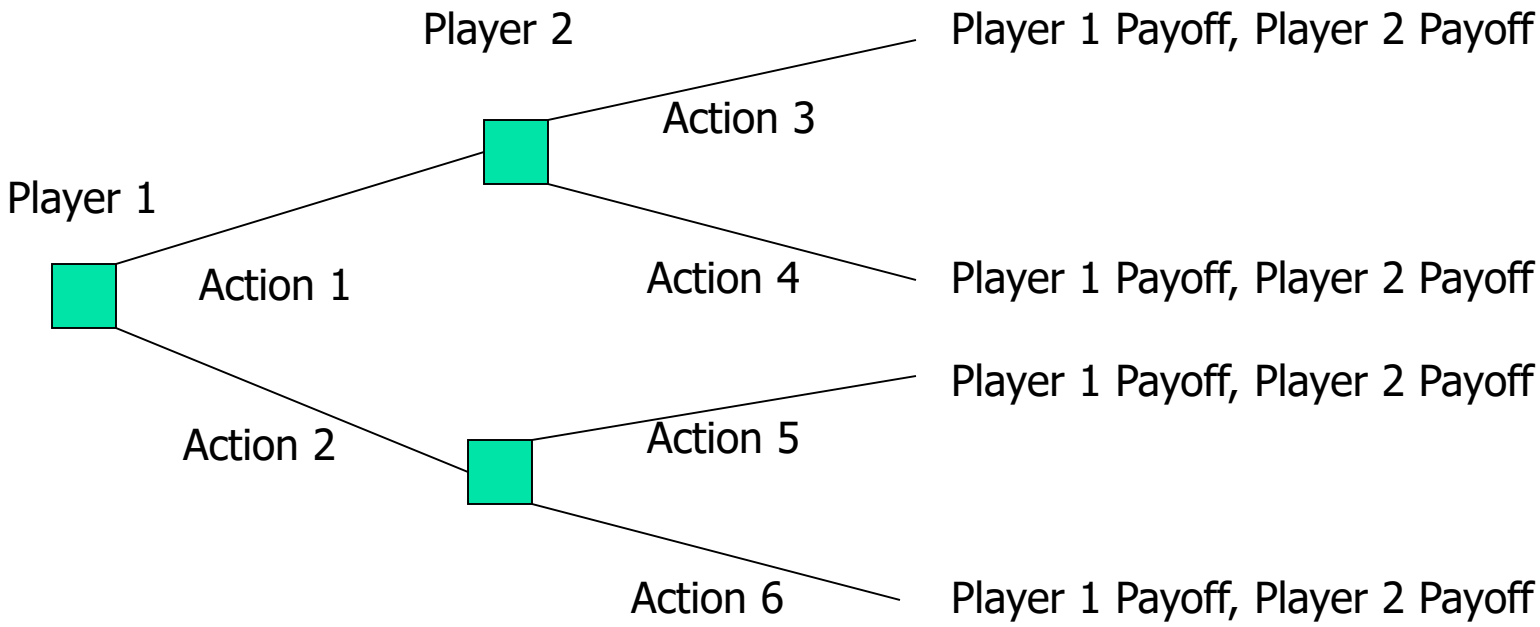
- A Prisoners' Dilemma occurs when all parties through noncooperative strategies obtain a less than optimal solution due to their self interest.
  - In the previous example, the Nash Equilibrium occurred at a sub-optimal solution for the game where farmer A chose large and Farmer B chose increase.
    - They both would have been better off by choosing small, decrease.



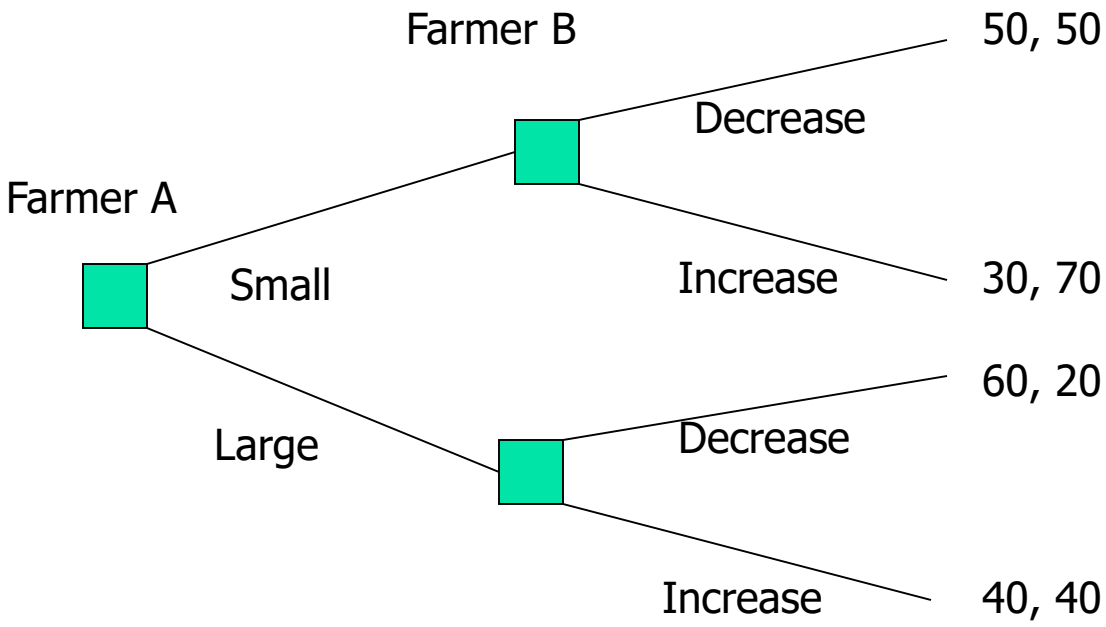
Game Trees

- A Game Tree is a way of representing a sequential move game.
- It is also known as the extensive form of a game.
- There are four components to a Game Tree.
  - Players
  - Payoffs
  - Nodes
    - A node represents a position within the game.
  - Actions
    - An action is a move that moves from one node to the next.

Representation of Game Tree



Game Tree Representation Using Previous Example Assuming Farmer A Moves First



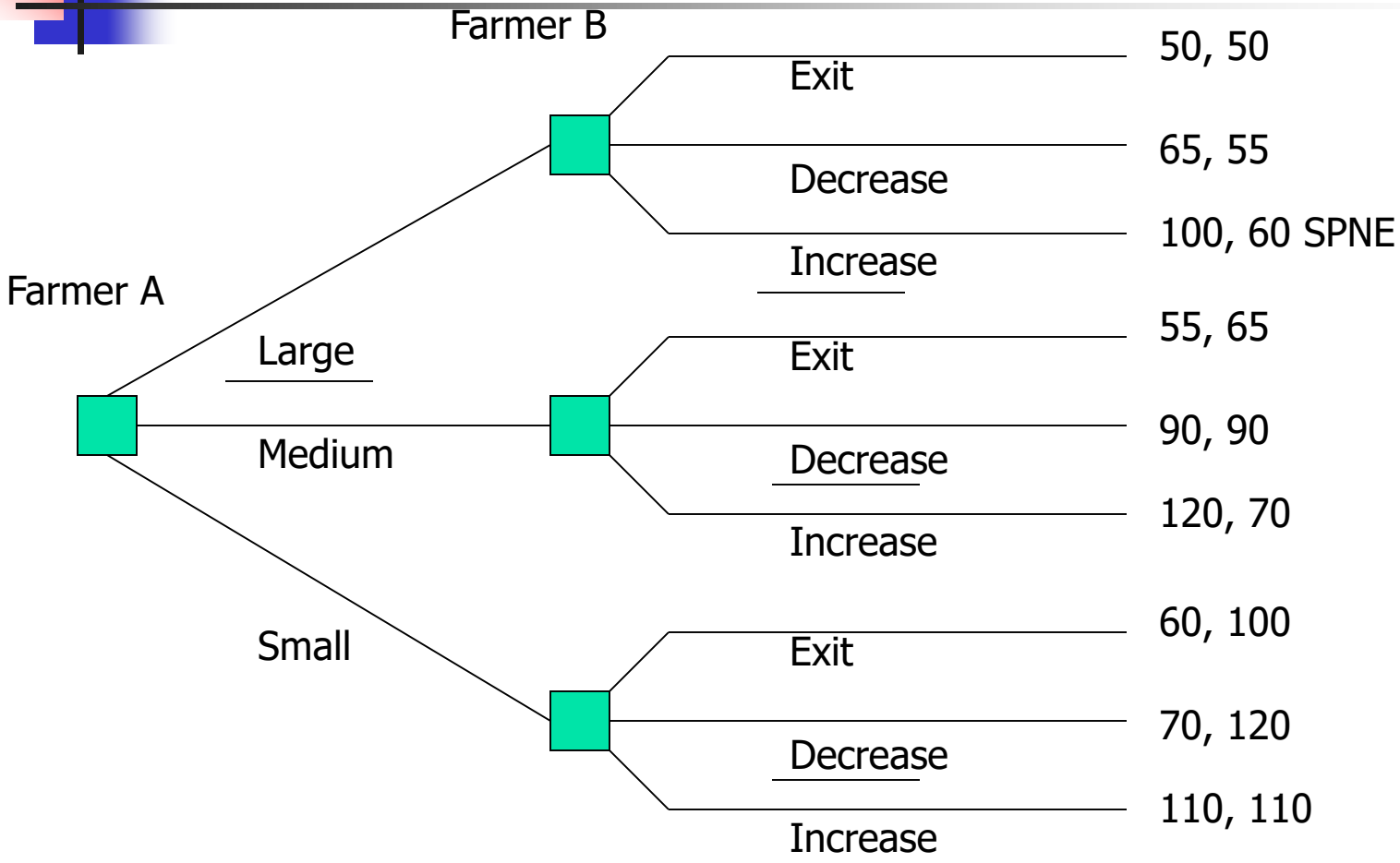
Subgame Perfection

- When working with a Game Tree, an important equilibrium concept is the Subgame Perfect Nash Equilibrium (SPNE).
- A SPNE is said to exist if "each player chooses an optimal action at each stage in the game that it might conceivably reach and believes that all other players will behave in the same way." (Besanko)

Subgame Perfection Cont.

- SPNE can be found by using a method called the rollback method.
  - In the rollback method, you start at the end of the tree and work your way back to find the best strategies for each node.
  - For our purposes, we can also call this a rollback equilibrium.

Subgame Perfection Example 2



Subgame Perfection Example 2

- The SPNE is where Farmer A becomes large and Farmer B increases.
- This gives a payoff of 100 to Farmer A and a payoff of 60 to Farmer B.
- If this was a simultaneous move game, what would the outcome had been?