

**Directions:** Please answer all items on this homework. You must show all your work. Unless otherwise stated, please simplify your answer.

1. Find the first order and second order (this includes the cross partial derivatives) partial derivatives of the following functions with respect to  $x_1$  and  $x_2$ : **(20 Points)**<sup>1</sup>
  - a.  $y = f(x_1, x_2) = 45x_1x_2$
  - b.  $y = f(x_1, x_2) = 4x_1^2 + 3x_1x_2 + 2x_2^2$
  - c.  $y = f(x_1, x_2) = x_1^{1/3}x_2^{2/3}$
  - d.  $y = f(x_1, x_2) = (x_1x_2)^{3/4}$
2. Use the first order conditions to find the critical points of the function. Use the second order conditions to show whether the critical points are maximum, minimum, or saddle points (point of inflection). Are the critical points relative or absolute extrema? Please explain. **(40 Points)**
  - a.  $y = f(x) = -4x^2 + 8x + 32$
  - b.  $y = f(x) = 6x^2 + 36x + 15$
  - c.  $y = f(x) = 2x^3 + 6x^2 + 6x + 2$
  - d.  $y = f(x) = -2x^3 + 24x + 45$
  - e.  $y = f(x) = -(x^2 - 4)^2$
3. Minimize  $f(x_1, x_2) = 4x_1 + 27x_2$  subject to the constraint  $300 = g(x_1, x_2) = x_1^{1/3}x_2^{2/3}$ . First solve this problem using the Lagrange Method. Next, resolve this problem by changing it into an unconstrained minimization problem. **(20 Points)**
4. Maximize  $f(x_1, x_2) = x_1^{2/3}x_2^{1/3}$  subject to the constraint  $16,200 = g(x_1, x_2) = 54x_1 + 8x_2$ . First solve this problem using the Lagrange Method. Next, resolve this problem by changing it into an unconstrained maximization problem. **(20 Points)**

---

<sup>1</sup> The cross derivatives for a function  $f(x_1, x_2)$  are  $\frac{\partial^2 y}{\partial x_1 \partial x_2}$  and  $\frac{\partial^2 y}{\partial x_2 \partial x_1}$