

General Instructions: This exam is worth **200 points**. You must provide your own paper. You are allowed one 3x5 note card for the exam. This note card can have anything on it but if it is larger than 3x5 you will get a zero on the exam. You are allowed to use a calculator. **You must show all your work when appropriate to get credit.** This includes showing all applicable formulas you use. No cell phones, music players (ipods), or other technology devices are allowed to be in your possession during the exam. If you are caught with any of these items, you will receive a zero on the exam. **(Good Luck!)**

Question 1 (100 Points Total): Suppose you have just been hired by Abraham Shepherd's Lambing Company, which is one of the largest suppliers of lambs to the local restaurant community. Your first task on the job is to figure out how many lambs you are going to produce this season and how many inputs you will need to purchase to produce your profit maximizing number of lambs. Your supervisor, Jacob Isaacson, informs you that the company has contracted a price of \$40 per lamb and that the restaurateurs will purchase all the lambs the company can produce given this price.

Since you are new to the company and you have very little experience with raising lambs, you decide to schedule a meeting with the production manager, David Christopher, who is in charge of raising the animals. In your meeting, you discover that there are four key feed items that are provided to the lambs beyond the pasture that they are raised on. These include durum wheat (D), urad beans (U), millet (M), and barley (B). You are told that these four items can be changed, while all the other inputs to production are fixed at certain quantities and have already been purchased for the season. These fixed inputs have been priced at \$7,520 and covers items like pastureland, water, labor, etc. The production manager also informs you that the company has a proprietary production function equation that has been developed over the years to estimate how many lambs can be produced by the variable inputs. This production function can be represented as:

$$L = f(D, U, M, B) = 4(9D^2 - U^2) + 6(3D+20U) - 2D^3 + 2(729M^3B^2)^{1/6};$$

where L is the number of lambs that are produced, D is the tons of durum wheat that is used, U is the tons of urad beans that are fed, M is the amount of millet measured in hundredweights (cwt), and B represents the amount of barley that is fed to the lambs, which is also measured in cwt.

Having finished your meeting with the production manager, you have decided to do an internet search on companies that sell each of these feed items. You have come across a company known as Sheepish Feeds. This company appears to offer the best prices for feed supplies among all the other suppliers that you have found. The company quotes you a price of \$720 per ton of durum wheat, \$960 per ton of urad beans, \$20 per cwt of millets, and \$60 per cwt of barley.

Please answer the following questions making sure to give proper justification:

- A) At the current given prices that Sheepish Feeds is quoting you and the restaurateurs will pay for the lambs, what is the optimal amount of profit you will receive for producing the optimal number of lambs? **(45 Points)**
- B) Suppose your supervisor comes to you and tells you that the herd of lambs have contracted a serious illness. Because of this, the lambs cannot eat urad beans, millet, or barley. He tasks you with figuring how many sheep need to be produced if the goal was to minimize average variable cost (AVC). He would also like to know how much profit he would lose if he produced at lowest AVC rather than the profit maximizing point based on this new condition. Hence, what is the lowest AVC that can be achieved and how much profit would he lose if he decided to minimize AVC rather than profit? You need to calculate a new profit max for this problem for both the AVC solution and the new profit max solution. **(25 Points)**
- C) Suppose your supervisor would like to produce 72 lambs using only Millet and Barley. Demonstrate to him what his cost minimizing solution would look like in graphical form, i.e., draw the cost minimizing solution on a graph. Assume the other two variables are zero and that fixed costs are zero. **(20 Points)**
- D) Suppose your supervisor would like to use only urad beans to produce lambs and no other inputs. What stage is the optimal solution in if you produced the profit maximizing amount of lambs under this new scenario? **(10 Points)**

Question 2 (80 Points Total): Suppose you work at Sheepish Feeds. Your boss has come to you to figure out how much the company should produce this year of millets and barley given that the company has 5,840 acres to devote to the two crops this year. The company would like to be very competitive this year so it has set a price for each of the products to be quite low. Specifically, the company would like to sell millets for \$20 per cwt and barley for \$60 per cwt. You are also told that the fixed land that you have available to produce these two crops costs \$15 per acre. You have other inputs to production, but they are not a limiting factor in production like acreage so you will treat the costs related to them as fixed costs. These fixed costs are \$25,200.

Over the years, your company has done an excellent job keeping track of its production data. The growing conditions are such that you can predict with fairly reasonable accuracy how many cwt's of barley and millet you can produce based on the amount of acres you allocate to each. You have found the following relationship between the acreage of land you allocate to millet and the production of millet: $M = f(A_M) = 9A_M^{1/3}$ where M is the quantity of millet produced in terms of cwt and A_M is the number of acres allocated to producing millet. Your analysis also shows that for the production of barley, there is the following relationship: $B = f(A_B) = 243A_B^{1/3}$ where B is the quantity of barley produced in terms of cwt and A_B is the number of acres allocated to growing barley.

Please answer the following questions based on the information given above:

- A) What is the optimal profit at your optimal solution? Note that if you use the optimality condition of setting the MVP's equal to each other to solve this question, you will lose 10 points on this question **(40 Points)**
- B) Graph the optimal solution. Be sure to use revenue rather than profit when you are graphing the optimal solution. **(30 Points)**
- C) Suppose management of the company wants to change the price of millets to \$80. How much would the price of barley need to be to give you the same output levels you found in part A. Please explain. **(10 Points)**

Question 3 (20 Points Total): Suppose there are two major feed companies that cater to the sheep industry. These two companies are Sheepish Feeds and Lamb Delights. Each of these company is a fierce competitor of the other. Hence, any decision made by one of the companies will have an effect on the other company and vice versa since the decision made by one can take clientele from the other. Currently, both companies are trying to figure their pricing strategies for the next lambing season.

Sheepish Feeds has four pricing strategies that it is considering for the next production season for lambs. Each of these strategies focuses on having the lowest price on particular items that it sells. Sheepish Feeds first strategy that it is considering is to have the lowest prices on all the different types of feeds it sells. This strategy is known as the Low Price on All Feeds (LPAF) strategy. The second strategy under consideration by the company is to have the lowest prices on all its feeds that are heavily grain based. This strategy is known as the Low Price on Grains Strategy (LPG). The company's third strategy is known as the Low Price on Protein Feeds (LPPF) strategy. This strategy calls for the company to sell all of its high protein formulation feeds at the lowest price possible. The last strategy Sheepish Feeds has is its Low Price on Milk Replacer Feeds. Under this strategy, the company markets its milk replacer feeds at the lowest price possible.

Lamb delights has three different pricing strategies that it is considering employing. Its first pricing strategy is known as the Two for One Sale (TOS) strategy. When following this strategy, the company will sell two bags of the same type of feed for the price of one bag. The second strategy that the company is considering is known as the Ten Percent Discount (TPD) strategy. This strategy requires the company to give a 10% discount over the retail price for all the feeds that it sells. The third strategy that Lamb Delights is considering is to have a price matching scheme, which is known as the Price Match (PM) strategy. For this strategy, the company vows to match any competitor's price on the feeds that it sells.

The table below represents the payoffs for each feed company in terms of weekly gross profits. You should assume that each company knows the potential strategies and payoffs of the other company and is trying to maximize its weekly profits. It should also be assumed that each company has to make its decision without knowing the decision of the other company. Sheepish Feeds' payoffs are represented first, while Lamb Delights' payoffs are second.

		Lamb Delights		
		Two for One Sale (TOS)	Ten Percent Discount (TPD)	Price Match (PM)
Sheepish Feeds	Low Price on All Feeds (LPAF)	375, 893	24, 952	25, 174
	Low Price on Grains (LPG)	622, 530	142, 181	174, 719
	Low Price on Protein Feeds (LPPF)	385, 165	452, 368	591, 176
	Low Price on Milk Replacer Feeds (LPMRF)	680, 463	305, 526	256, 760

Please answer the following questions:

A) Are there any dominant or dominated strategies for either feed companies? If so, what is it or are they? **(5 Points)**

B) Does a Nash equilibrium exist? If so, what is it or are they? **(5 Points)**

C) If Lamb Delights could get advanced knowledge of the pricing strategy Sheepish Feeds will employ before the company makes its decision, which pricing strategy would each decide to employ based on the idea of a Subgame Perfect Nash Equilibrium? Please explain. **(10 Points)**