SYLLABUS FOR THE WRITTEN EXAMINATION IN ALGEBRA

The examination covers basic properties of fundamental algebraic structures. The following is a (not necessarily exhaustive) list of topics from which questions are drawn.

1. LINEAR ALGEBRA
   - Vector Spaces
     subspaces, bases, dimension, direct sums
   - Linear Transformations and Matrices
     invertibility, matrix representation of a linear transformation, determinants, similarity, eigenvalues and eigenvectors, diagonalization

2. GROUP THEORY
   - Groups
     basic properties of groups, subgroups, Lagrange’s Theorem, direct products, normal subgroups, quotient groups, cyclic groups, permutation groups, simple groups (definition of; simplicity of $A_n$ for $n \geq 5$), Cayley’s Theorem, Fundamental Theorem of Finite Abelian Groups
   - Homomorphisms
     kernel, image, isomorphisms, isomorphism theorems, automorphisms

3. RING THEORY
   - Rings
     basic properties of rings, subrings, ideals, quotient rings, ring homomorphisms, isomorphism theorems, direct sums
   - Integral Domains and Polynomial Rings
     PID’s, irreducible, prime, units, associates, UFD’s, Euclidean domains, division algorithm, criteria for irreducibility
   - Fields
     characteristic, construction via quotient rings, fields as vector spaces, polynomial ring over a field

The material is found in a large number of texts, and is approached in a rather uniform fashion. Some texts that have been recently used are:

Linear Algebra
Axler, *Linear Algebra Done Right*
Friedberg, Insel and Spence, *Linear Algebra*
Lang, *Linear Algebra*

Group and Ring Theory
M. Artin, *Algebra*
Dummit and Foote, *Abstract Algebra*
Fraleigh, *A First Course in Abstract Algebra*
Gallian, *Contemporary Abstract Algebra*
Herstein, *Abstract Algebra*